



UNIVERSITY OF AMSTERDAM

heat transport in tissue
bioheat equation
thermal properties
vascular cooling

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Thermal therapy

Hyperthermia

- Heating tumor to 40-43°C for 1h
- Combined with radiotherapy and/or chemotherapy
- Tumor-selective radiosensitization

Thermal therapy

Hyperthermia

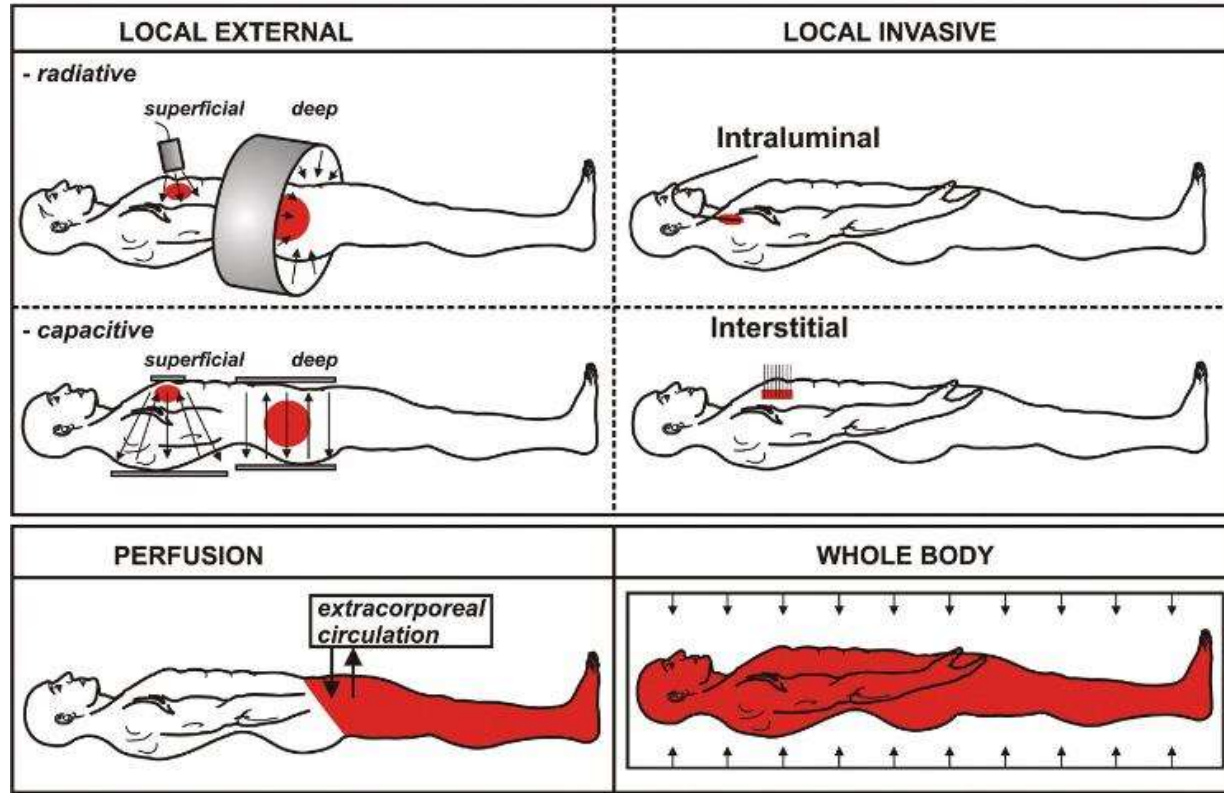
- Heating tumor to 40-43°C for 1h
- Combined with radiotherapy and/or chemotherapy
- Tumor-selective radiosensitization

Thermal ablation

- Heating tumor to 80-100°C for few minutes
- Direct tissue ablation

Thermal therapy

HYPERTHERMIA TECHNIQUES

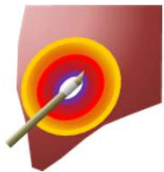


Heated volume differs strongly

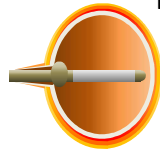
Thermal therapy

Local heating

Interstitial Hyperthermia /
Thermal ablation



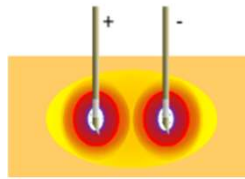
Intraluminal
hyperthermia/
HIVEC



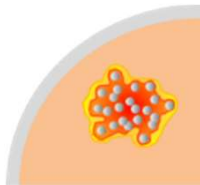
Superficial Hyperthermia



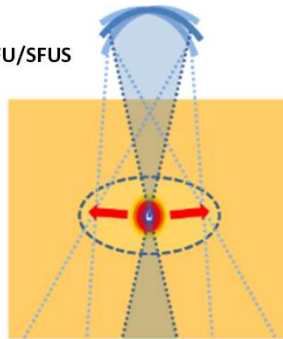
Electroporation



Ferromagnetic seeds/
Nanoparticles



HIFU/SFUS



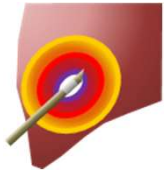
Heated volume
differs strongly

- 1-4 cm

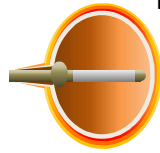
Thermal therapy

Local heating

Interstitial Hyperthermia / Thermal ablation



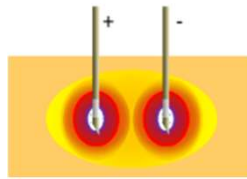
Intraluminal hyperthermia/ HIVEC



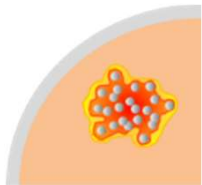
Superficial Hyperthermia



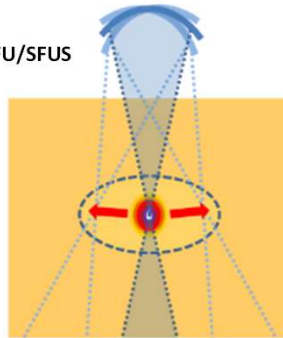
Electroporation



Ferromagnetic seeds/ Nanoparticles

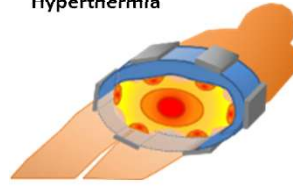


HIFU/SFUS

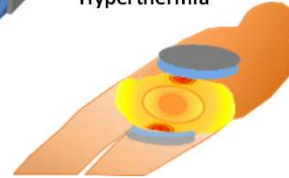


Regional heating

Radiative Loco-regional Hyperthermia



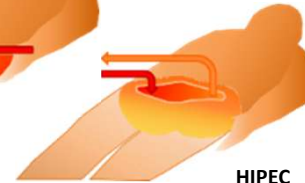
Capacitive Loco-regional Hyperthermia



Isolated perfusion



HIPEC



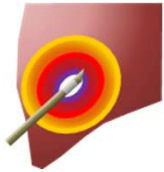
Heated volume differs strongly

- 1-4 cm
- 10-20 cm

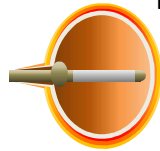
Thermal therapy

Local heating

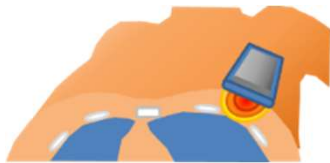
Interstitial Hyperthermia / Thermal ablation



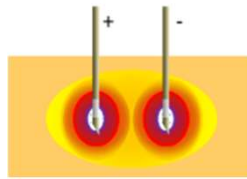
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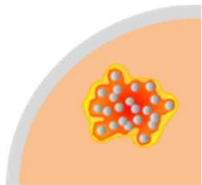
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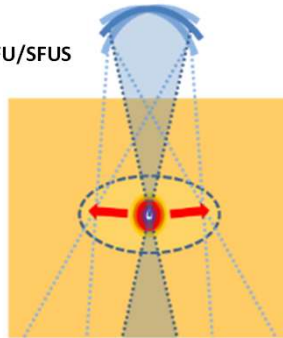
Electroporation



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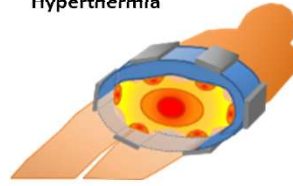


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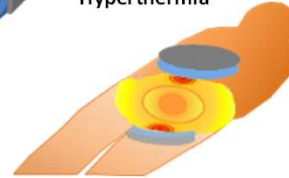


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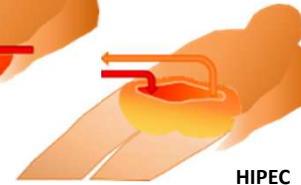
Radiative Loco-regional Hyperthermia



Capacitive Loco-regional Hyperthermia

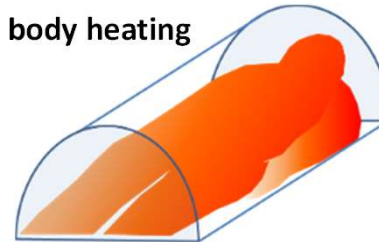


Isolated perfusion



HIPEC

Whole body heating



Heated volume differs strongly

- 1-4 cm
- 10-20 cm
- Whole body

Modeling heat transport in tissue



Modeling heat transport in tissue

- Bio-heat equation
- Thermal properties of tissues
- Vascular cooling



Modeling heat transport in tissue

- Bio-heat equation
 - Different models used
 - Pennes bio heat equation
 - Effective conductivity
 - Discrete vessels



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Modeling heat transport in tissue

- Pennes bio-heat equation
 - Describes heat transport in tissue by
 - Conduction
 - Blood flow

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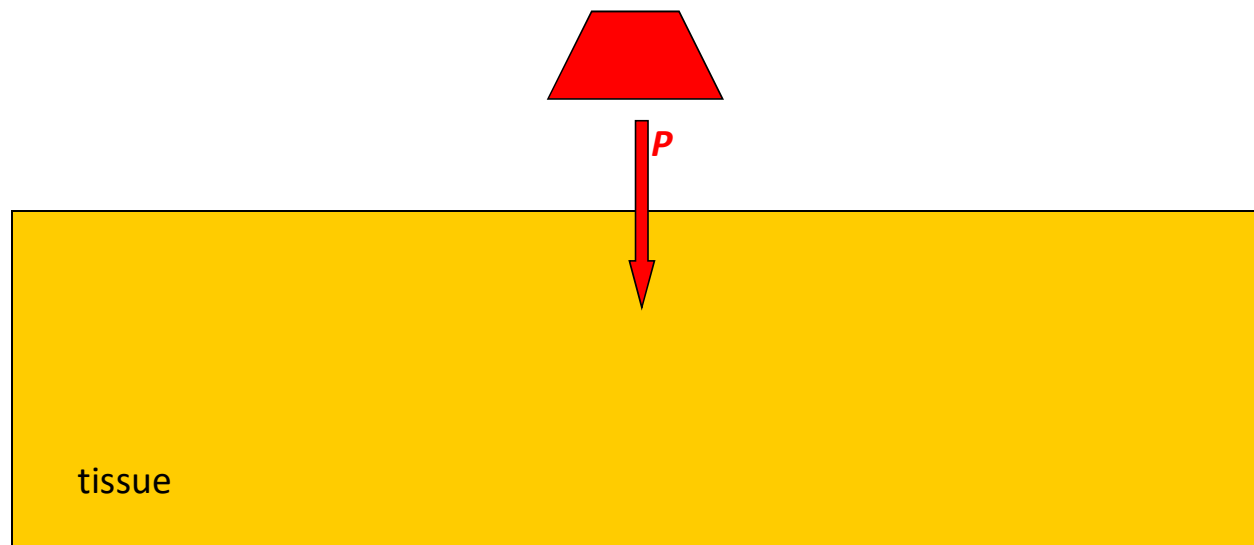
Heat transport by conduction

- A volume of non-perfused material



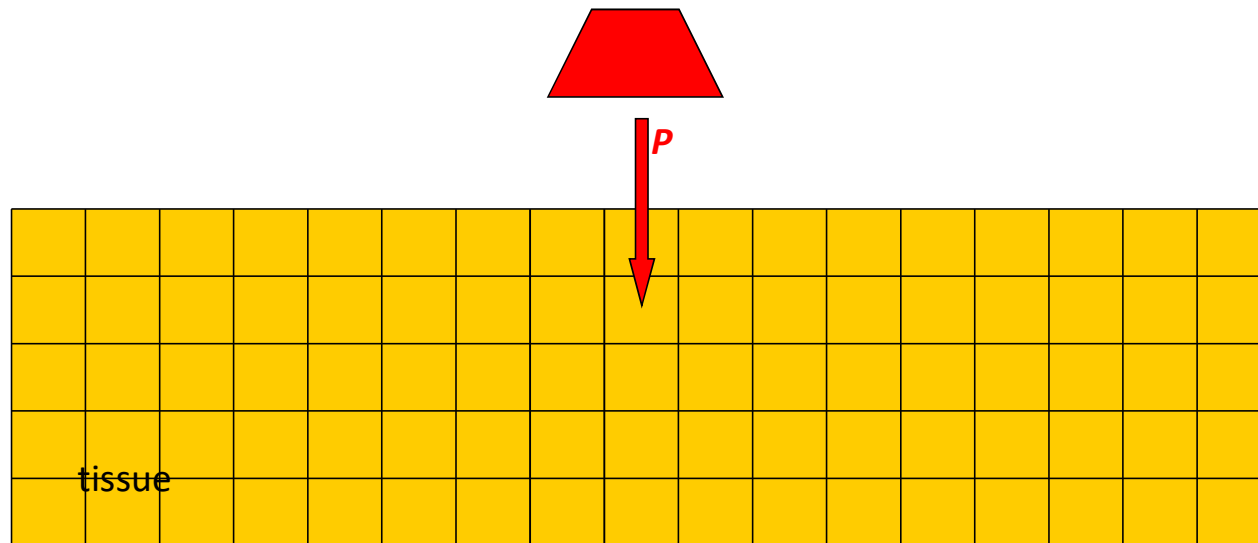
Heat transport by conduction

- Applicator depositing power



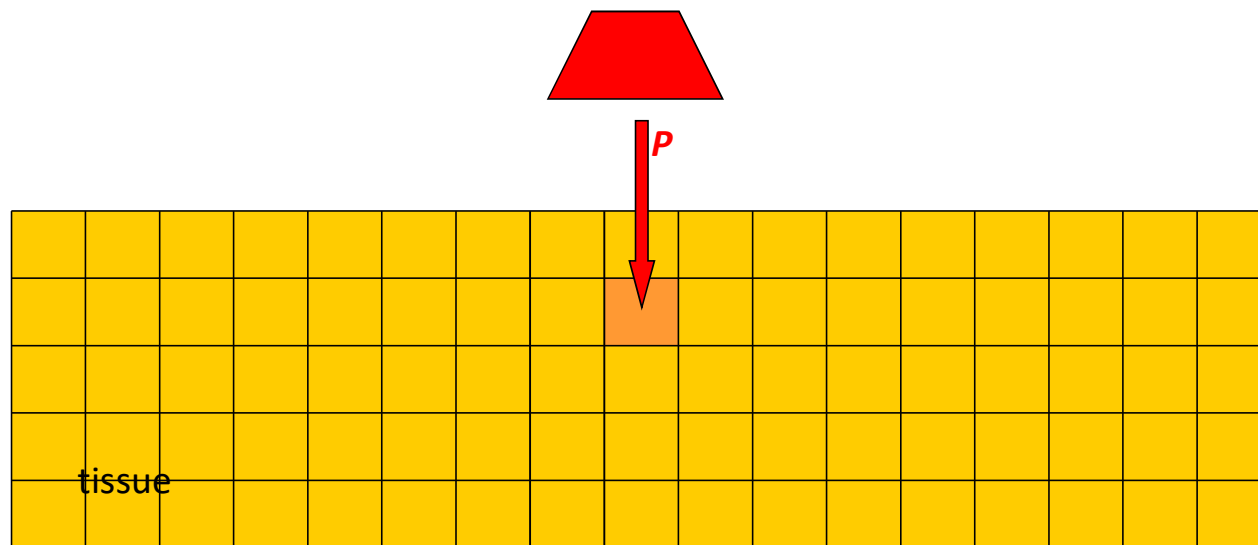
Heat transport by conduction

- Division into subvolumes: voxels



Heat transport by conduction

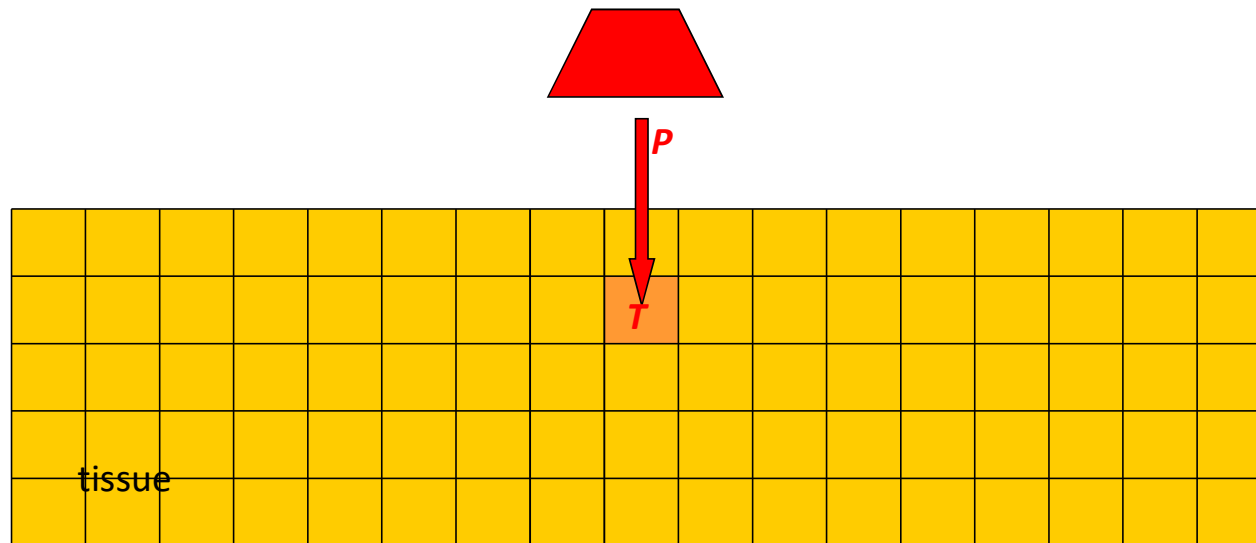
- heat balance for one voxel



Heat transport by conduction

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + P$$

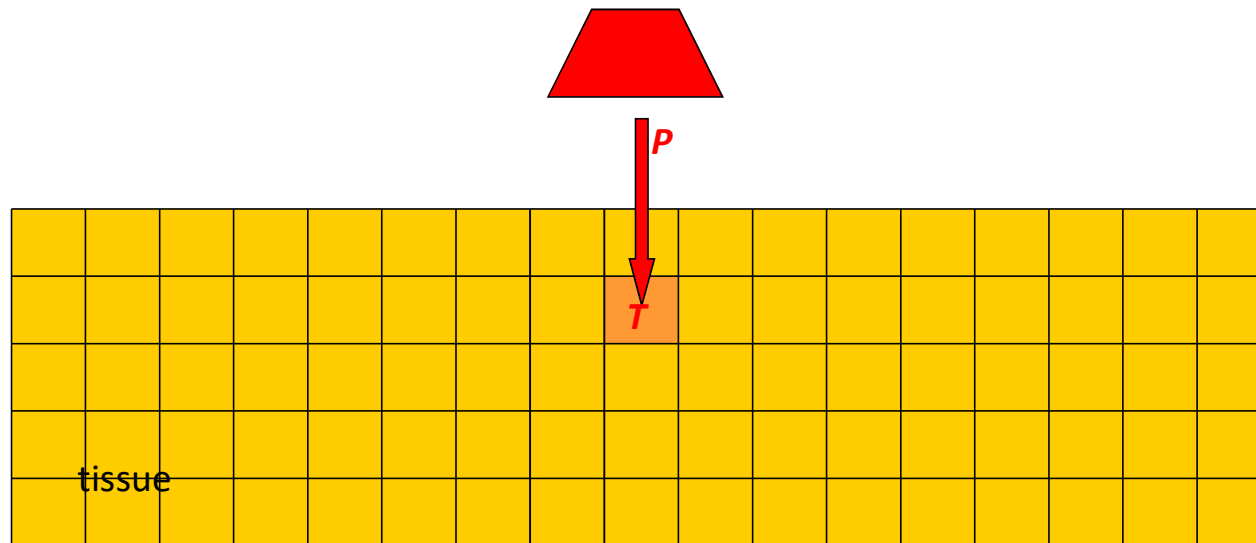
Energy balance equation



Heat transport by conduction

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + P$$

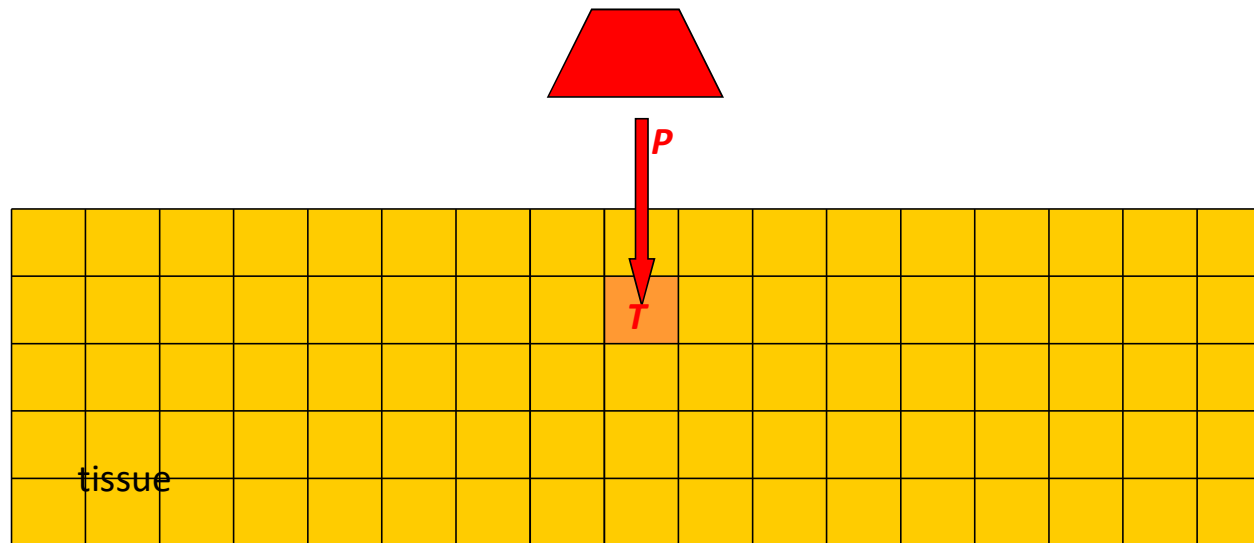
- Temperature change $c\rho \frac{\partial T}{\partial t}$
 ρ tissue density
 c tissue heat capacity



Heat transport by conduction

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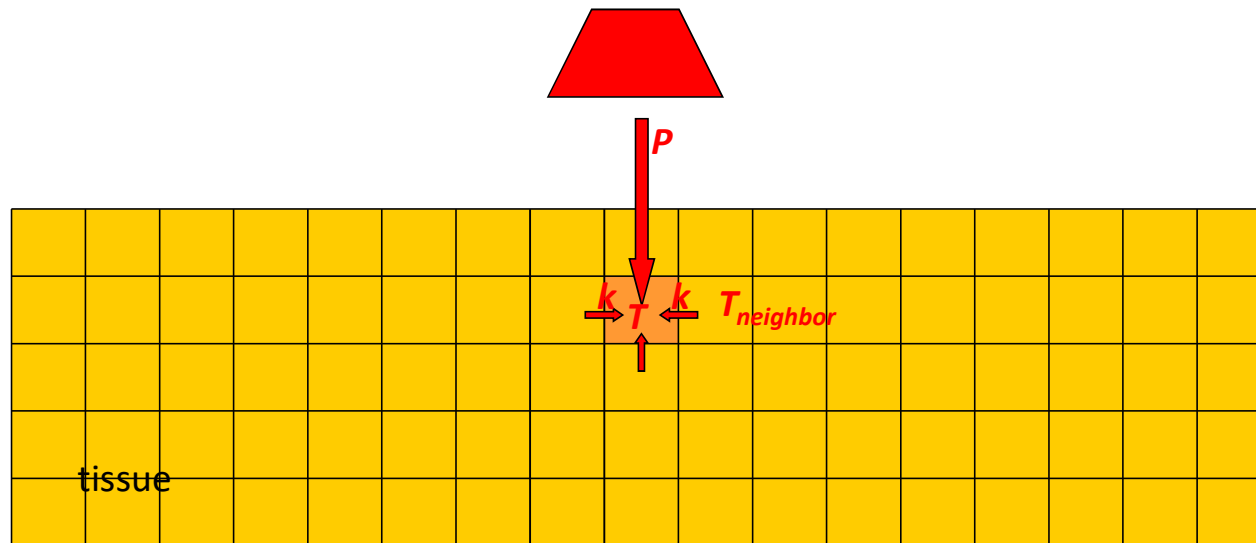
- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P



Heat transport by conduction

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + P$$

- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P
- Conduction $\nabla \cdot (k\nabla T)$
 k tissue conductivity

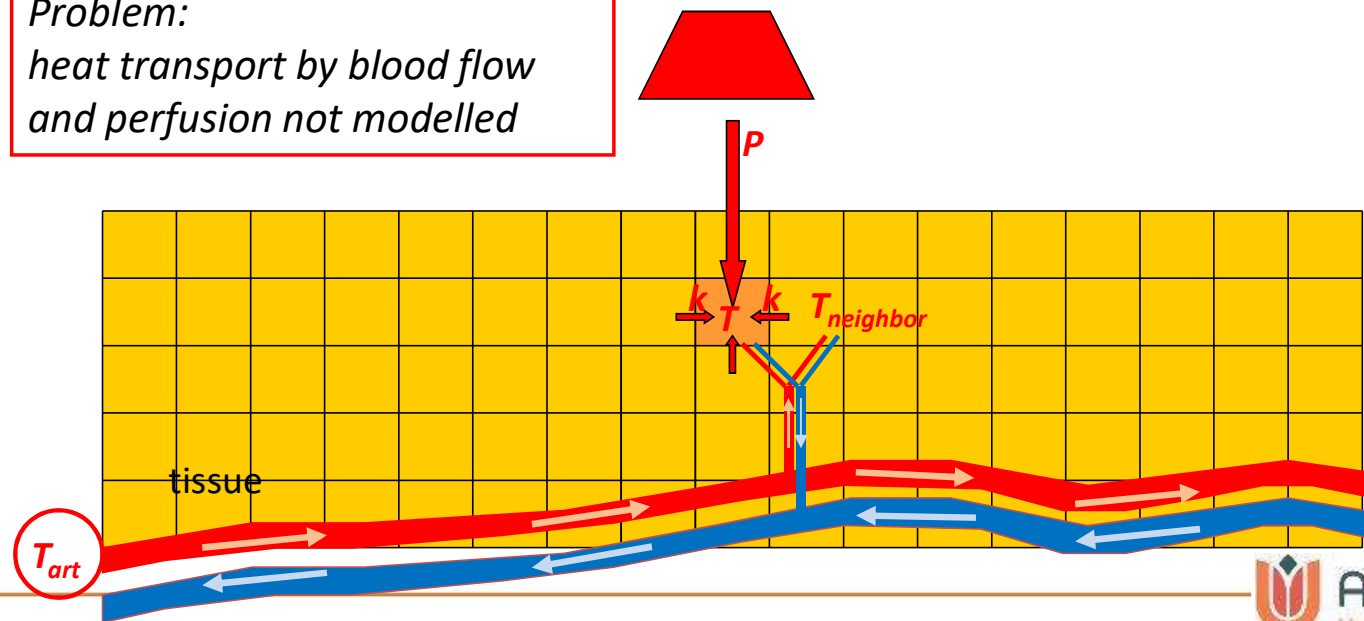


Heat transport by conduction

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + P$$

- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P
- Conduction $\nabla \cdot (k\nabla T)$

*Problem:
heat transport by blood flow
and perfusion not modelled*



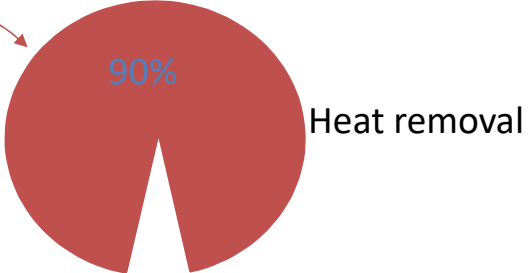
Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) - c_b W_b (T - T_{art}) + P$$

The diagram illustrates the Pennes bio heat equation. The equation is shown with two terms circled in red: $\nabla \cdot (k\nabla T)$ and $-c_b W_b (T - T_{art})$. A red arrow points from the first term to a pie chart, and another red arrow points from the second term to the same pie chart. The pie chart is labeled "Heat removal" and shows that 90% of heat removal is due to perfusion (represented by a red slice) and 10% is due to conduction (represented by a smaller red slice).

Perfusion is the dominant heat removal mechanism

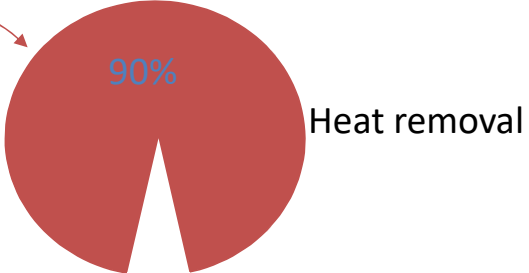
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90% Heat removal

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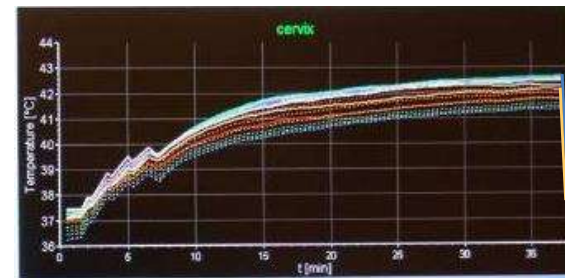
$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$


90% Heat removal

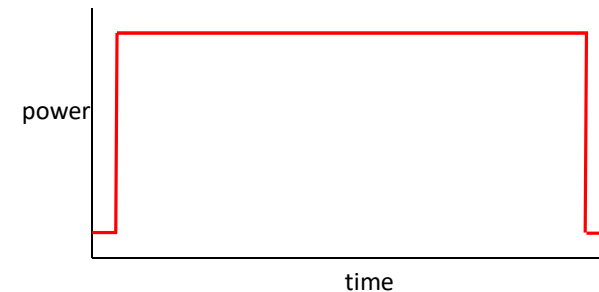
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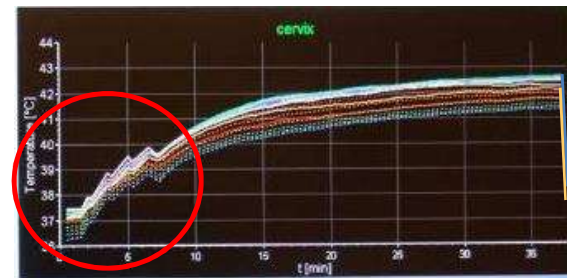


This equation simplifies interpretation of data:



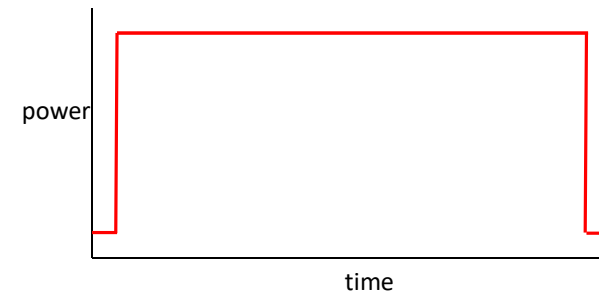
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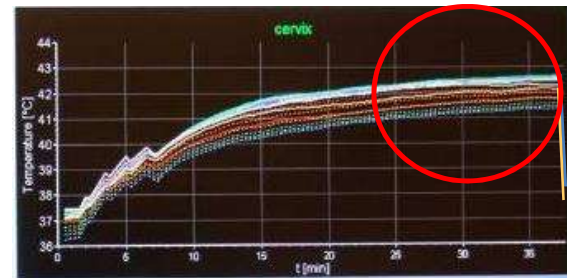
This equation simplifies interpretation of data:

- At the start of treatment



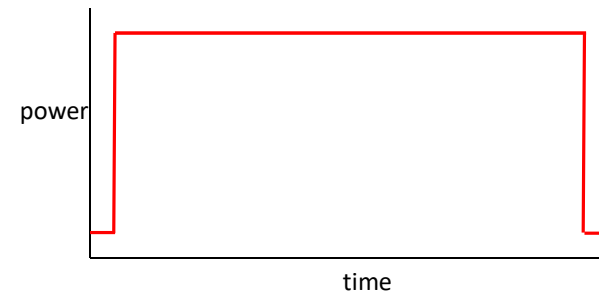
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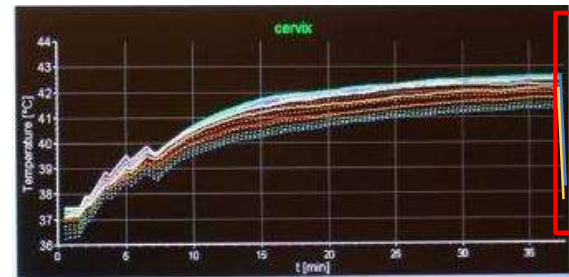
This equation simplifies interpretation of data:

- At the start of treatment
- During steady state



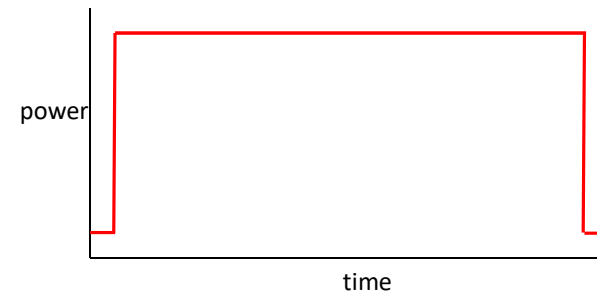
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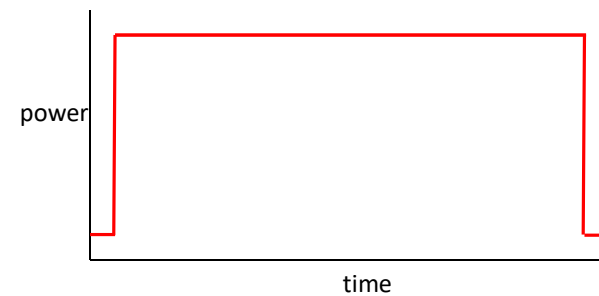
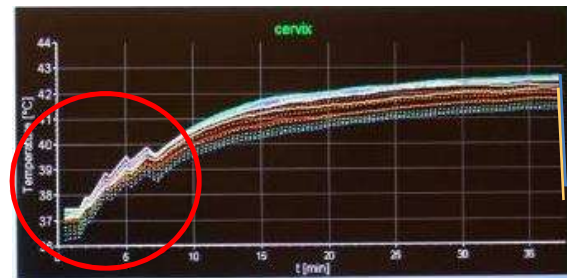
- At the start of treatment
- During steady state
- After power off



Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$

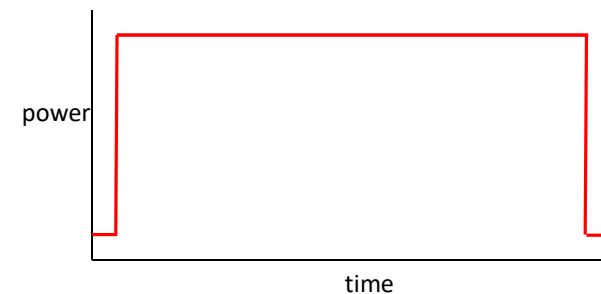
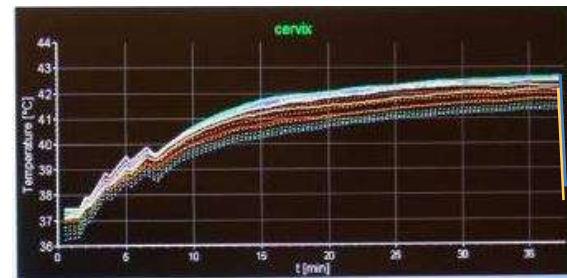
At the start of treatment $T = T_{art}$:



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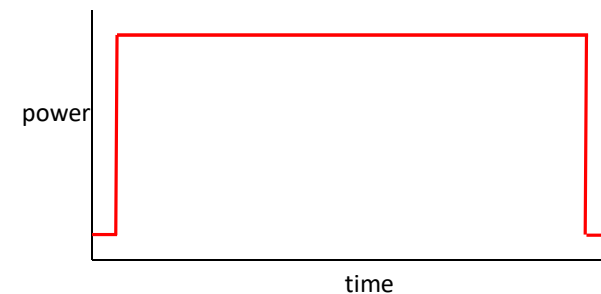
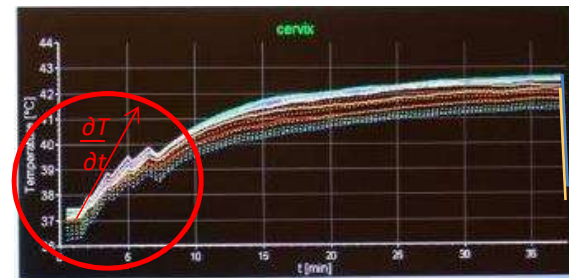
$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$

At the start of treatment $T = T_{art}$:

$$c\rho \frac{\partial T}{\partial t} = P$$

At start of treatment:

rate of temperature rise is proportional to absorbed power P



Pennes bio heat equation

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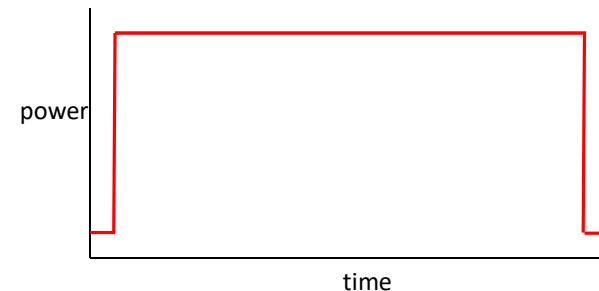
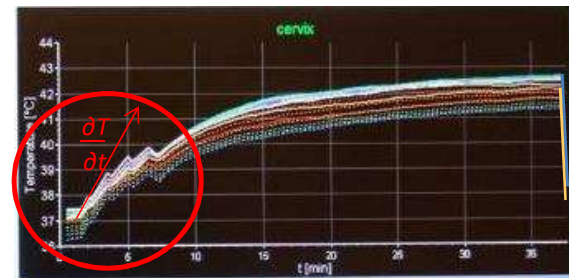
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Temperature rise after 60 sec power pulse indicative for P distribution

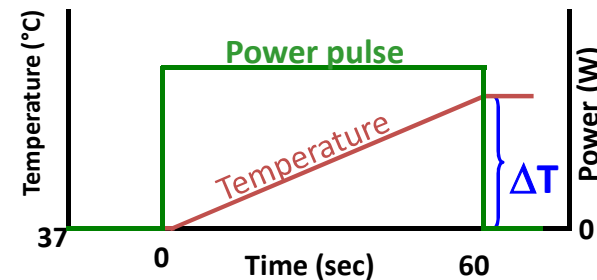


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Temperature rise after 60 sec power pulse indicative for P distribution

Pennes bio heat equation

*Power pulse
procedure prescribed
in 1989 ESHO
QA guidelines*

6. Characterization of applicators

6.1. The effective field size (EFS) of an applicator is defined by the 50% SAR contour measured at a depth of 10 mm from the surface of a plane homogeneous phantom with the dielectric properties of muscle. The penetration depth (PD) is defined as the distance below 10 mm at which the SAR is 50% of that at 10 mm depth. Measurements to determine the penetration depth must be made from the position of maximum SAR at 10 mm depth.

6.2. Both EFS and PD must be measured with the applicator arranged as the 'clinical set-up', including a bolus if appropriate. If a bolus is used, then its temperature should be equal to the initial temperature of the phantom.

6.3. EFS and PD should be determined by measuring the changes in temperature resulting from a brief pulse of high power. If non-perturbing E-field probes, thermographic imaging or liquid-crystal sheet imaging are used to determine SAR distributions, such measurements should be corroborated by measurements obtained using the power pulse technique.

6.4. If either a radiofrequency capacitive technique or a multi-element array of applicators is used, the EFS can depend critically on the particular geometry involved. Additional characterization of these techniques should be attempted using geometrically realistic phantoms.

6.5. When using the power pulse technique, measurements to determine the EFS and PD must be made within 60 seconds of the start of the pulse to minimize artefacts caused by thermal conduction within the phantom.

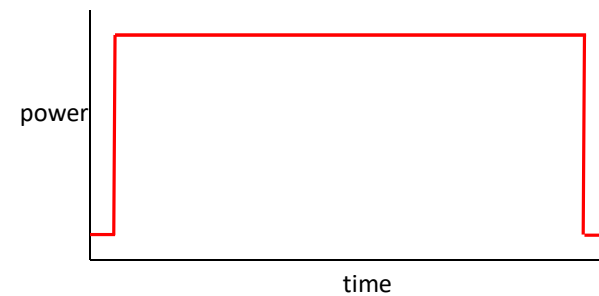
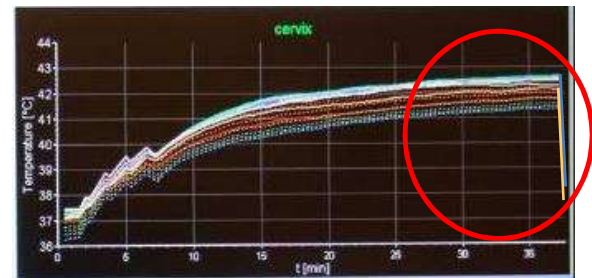
6.6. The plane muscle phantom used for determining EFS and PD must be 10 cm thick and must extend at least 5 cm beyond the physical dimensions of the applicator and bolus. In the case of radiofrequency capacitive electrodes, the thickness of the phantom should be equal to that of the tissue between the electrodes in the clinical set-up.

Temperature rise after 60 sec power pulse indicative for P distribution

Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$

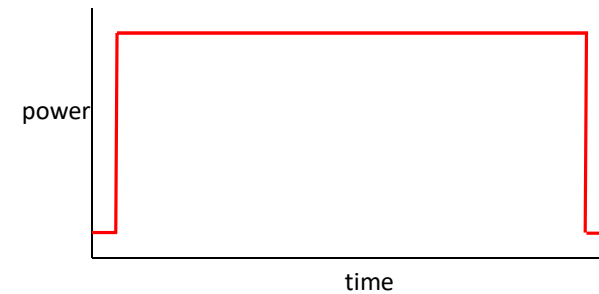
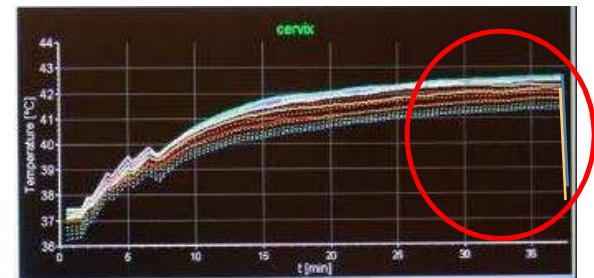
At steady state $c\rho \frac{\partial T}{\partial t} = 0$:



Pennes bio heat equation

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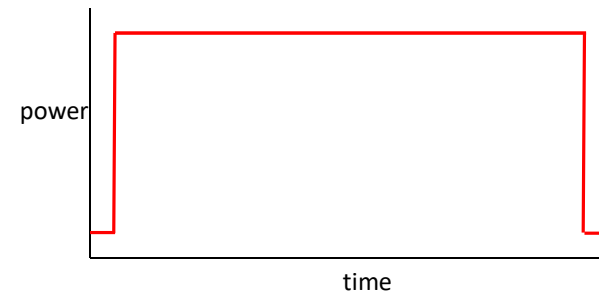
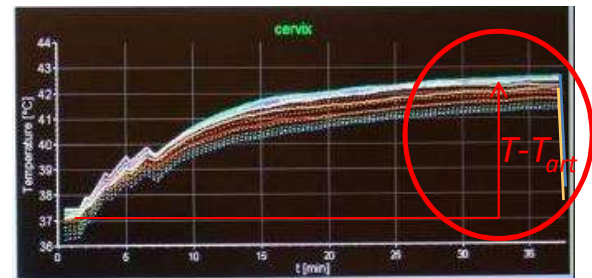
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During steady state:

Temperature rise $T - T_{art}$ is proportional to absorbed power P



Pennes bio heat equation

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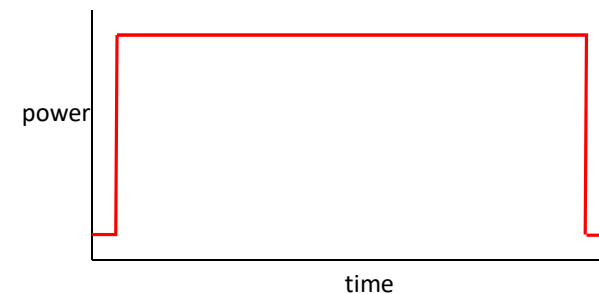
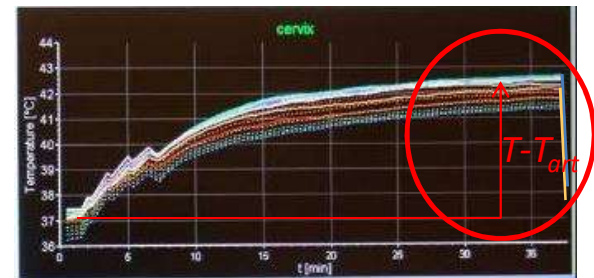
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Absorbed power distribution indicative for temperature distribution



Pennes bio heat equation

*measurement of
Absorbed Power
distribution
prescribed
to characterise
applicators in
1989 ESHO
QA guidelines*

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6.5. When using the power pulse technique, measurements to determine the EFS and PD must be made within 60 seconds of the start of the pulse to minimize artefacts caused by thermal conduction within the phantom.

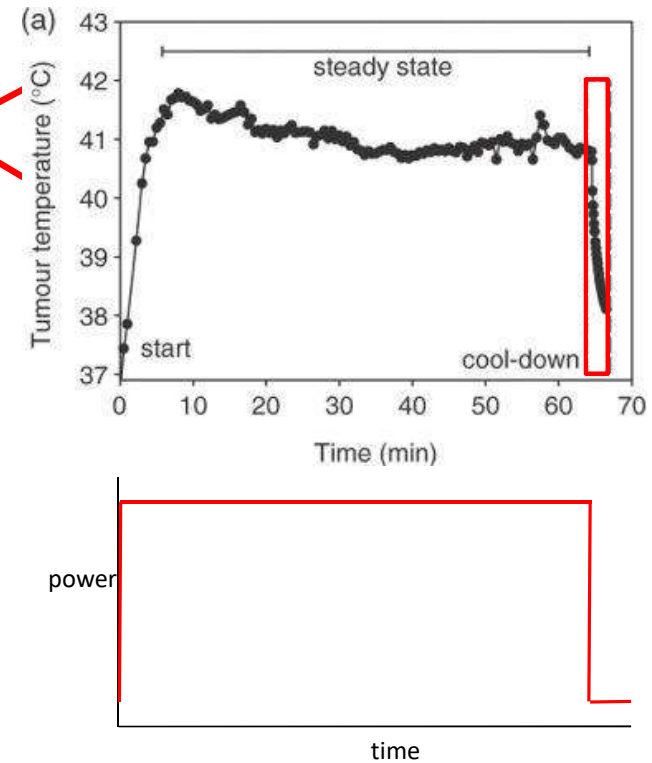
6.6. The plane muscle phantom used for determining EFS and PD must be 10 cm thick and must extend at least 5 cm beyond the physical dimensions of the applicator and bolus. In the case of radiofrequency capacitive electrodes, the thickness of the phantom should be equal to that of the tissue between the electrodes in the clinical set-up.

Absorbed power distribution indicative for temperature distribution

Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$

After power off: $P = 0$

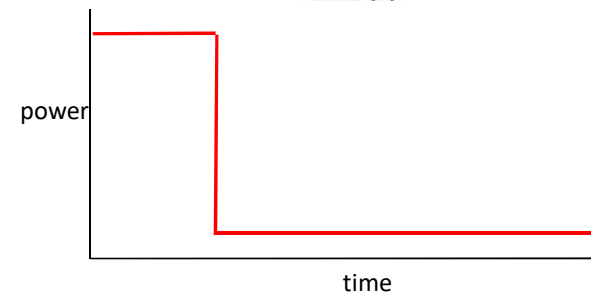
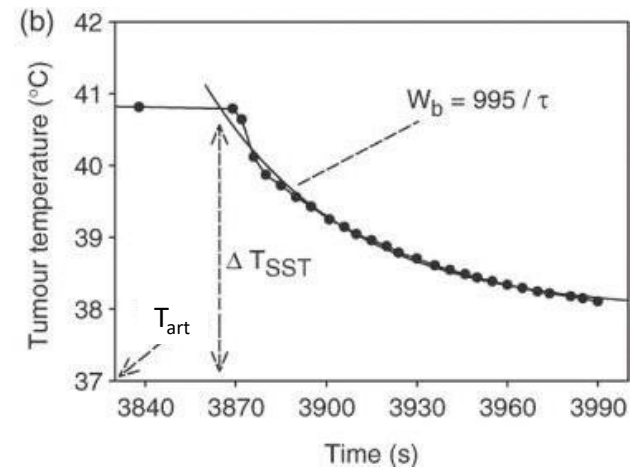


Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art})$$

fit an exponential function:

$$T(t) = T_{art} + \Delta T_{SST} \cdot \exp(-t/\tau)$$



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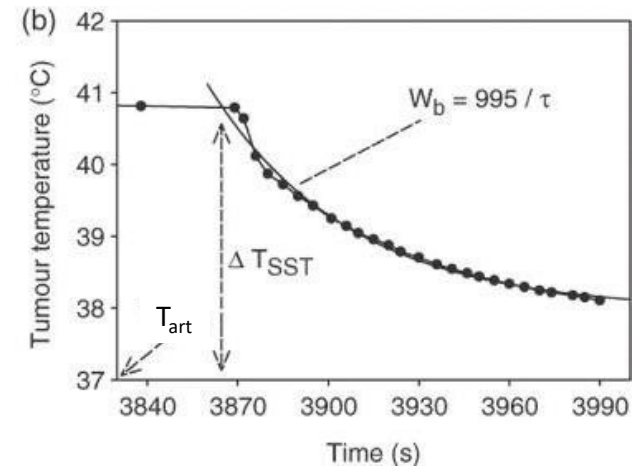
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yields:

$$\rho_t c_t \cdot \frac{-\Delta T_{SST}}{\tau} \cdot \exp(-t/\tau) = -c_b W_b \cdot \Delta T_{SST} \cdot \exp(-t/\tau)$$

$$W_b = \frac{\rho_t c_t}{c_b \tau}$$



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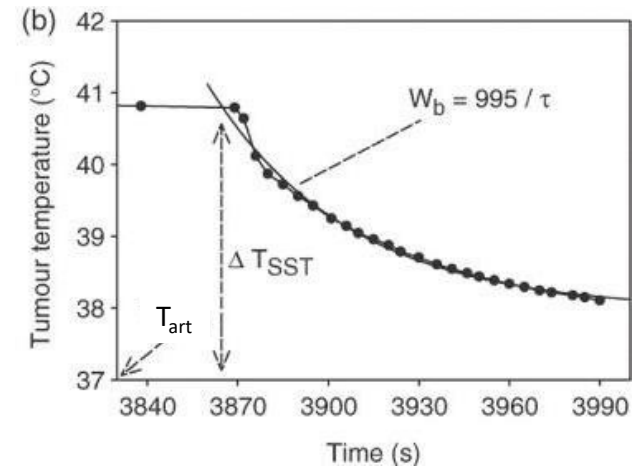
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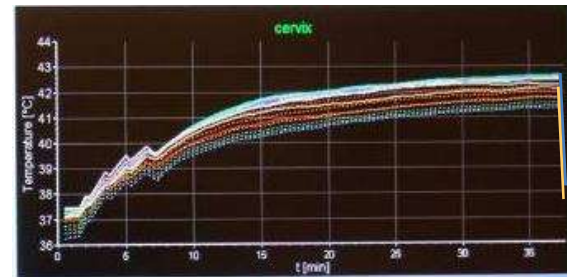
$$W_b = \frac{\rho_t c_t}{c_b \tau}$$

Perfusion W_b can be derived from decay time



Pennes bio heat equation

$$c\rho \frac{\partial T}{\partial t} = -c_b W_b (T - T_{art}) + P$$

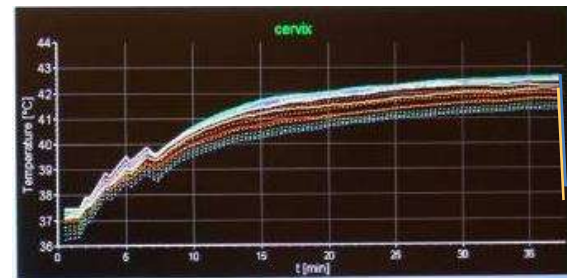


This simple equation allows us to derive parameters from measured T data:

- power density P
- tissue perfusion W_b

Pennes bio heat equation

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This simple equation allows us to derive parameters from measured T data:

- power density P
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But is this correct?

Heat transport in tissue

- Bio-heat equation
 - Different models used
 - Pennes bio heat equation
 - Effective conductivity
 - Discrete vessels

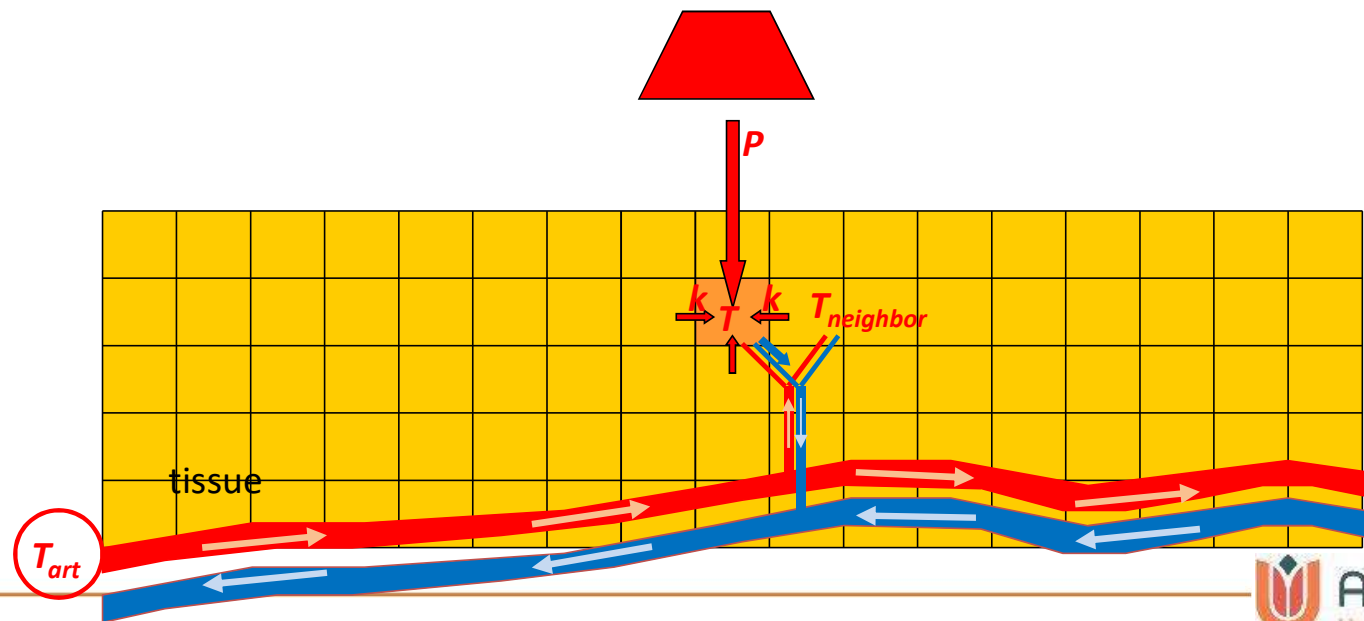
Heat transport in tissue

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Heat transport in tissue

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) - c_b W_b (T - T_{art}) + P$$

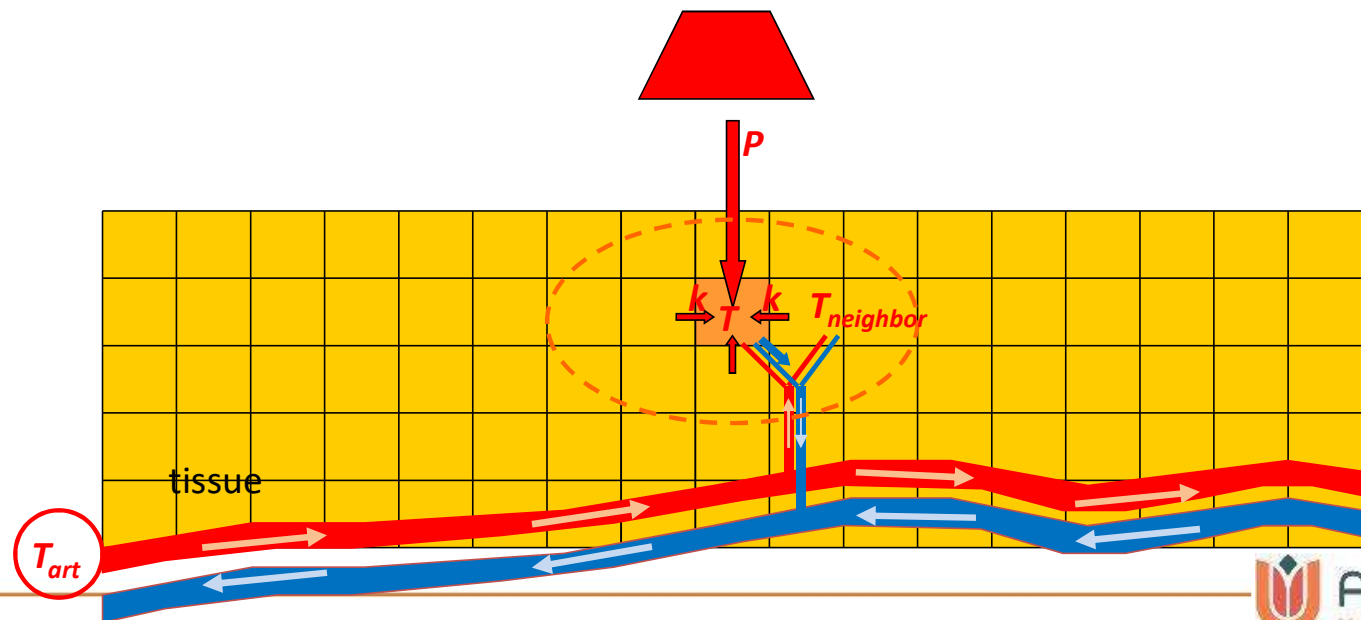
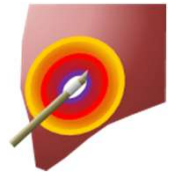
- $T_{art} = 37^\circ\text{C}$



Heat transport in tissue

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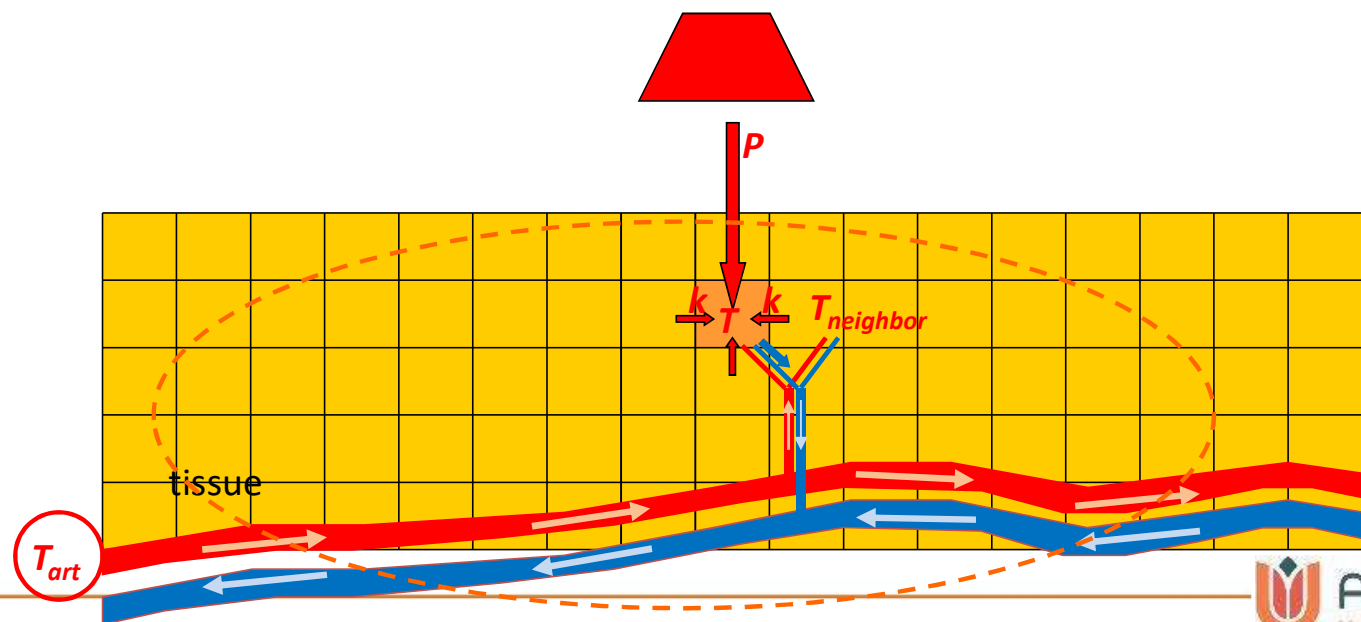
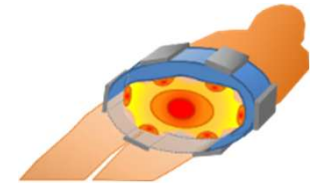
- $T_{art} = 37^\circ\text{C}$
- Validity assumption:
 - small heated volume 😊



Heat transport in tissue

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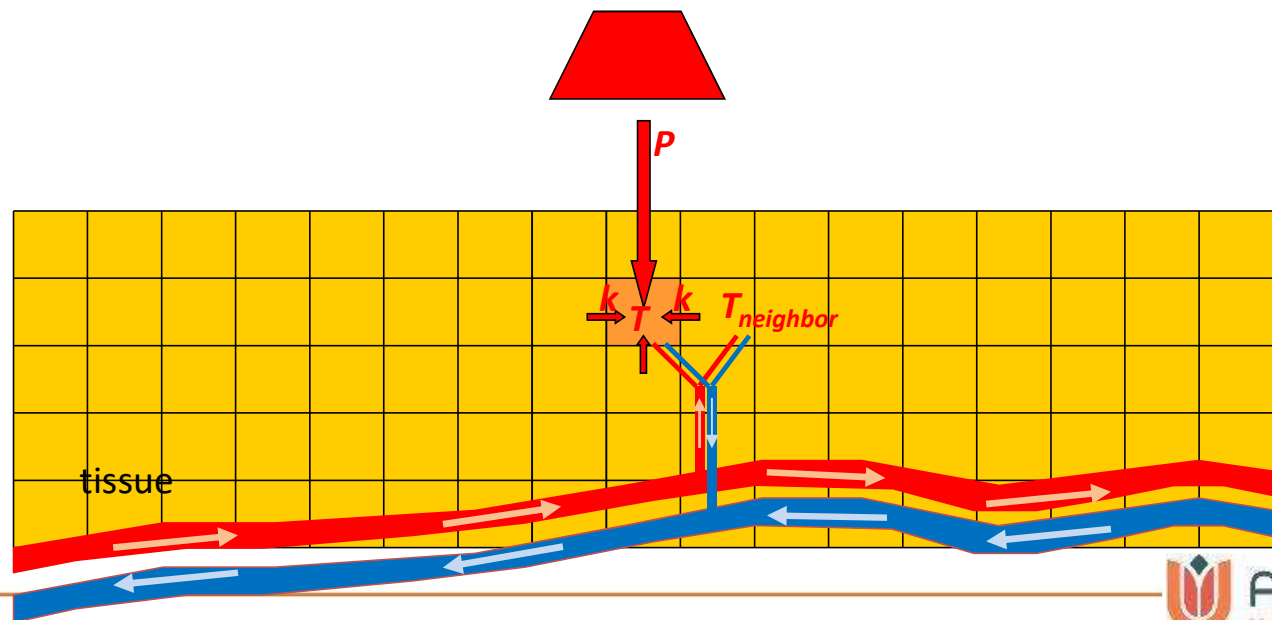
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 - large heated volume ☹



Heat transport in tissue

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + P$$

- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P
- Conduction $\nabla \cdot (k\nabla T)$



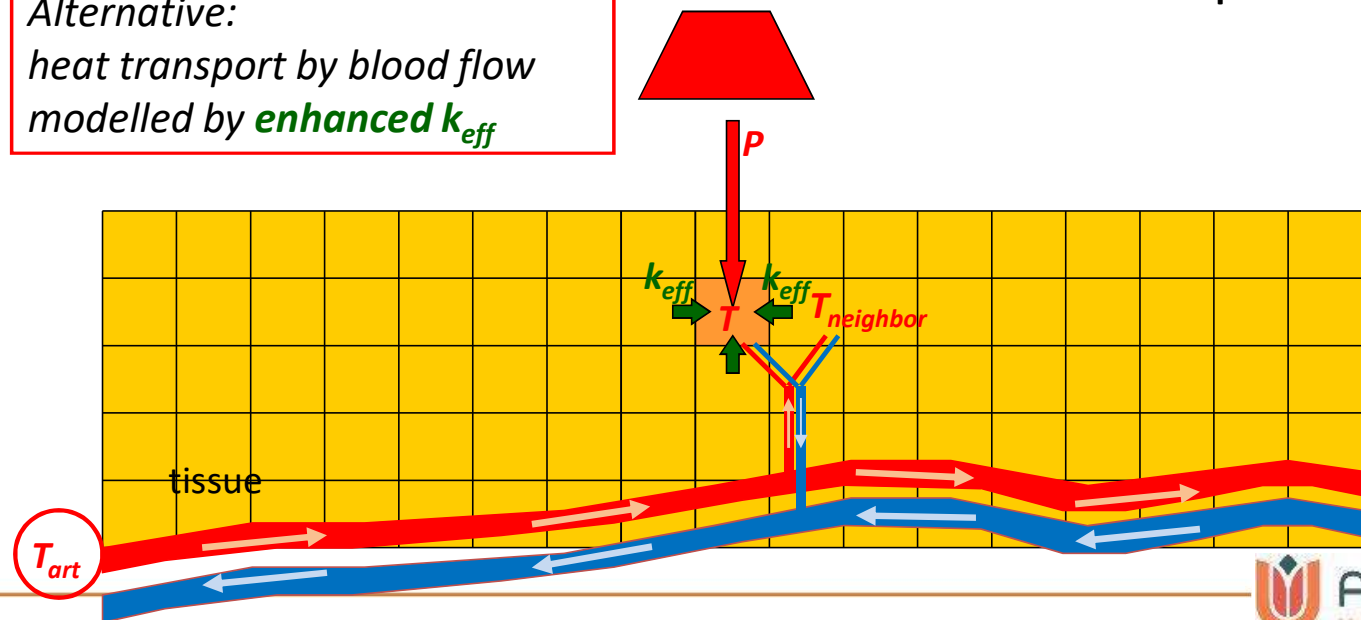
Heat transport in tissue

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k_{eff} \nabla T) + P$$

- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P
- Conduction $\nabla \cdot (k_{eff} \nabla T)$

Represents **conduction**
and **perfusion**

Alternative:
heat transport by blood flow
modelled by **enhanced k_{eff}**



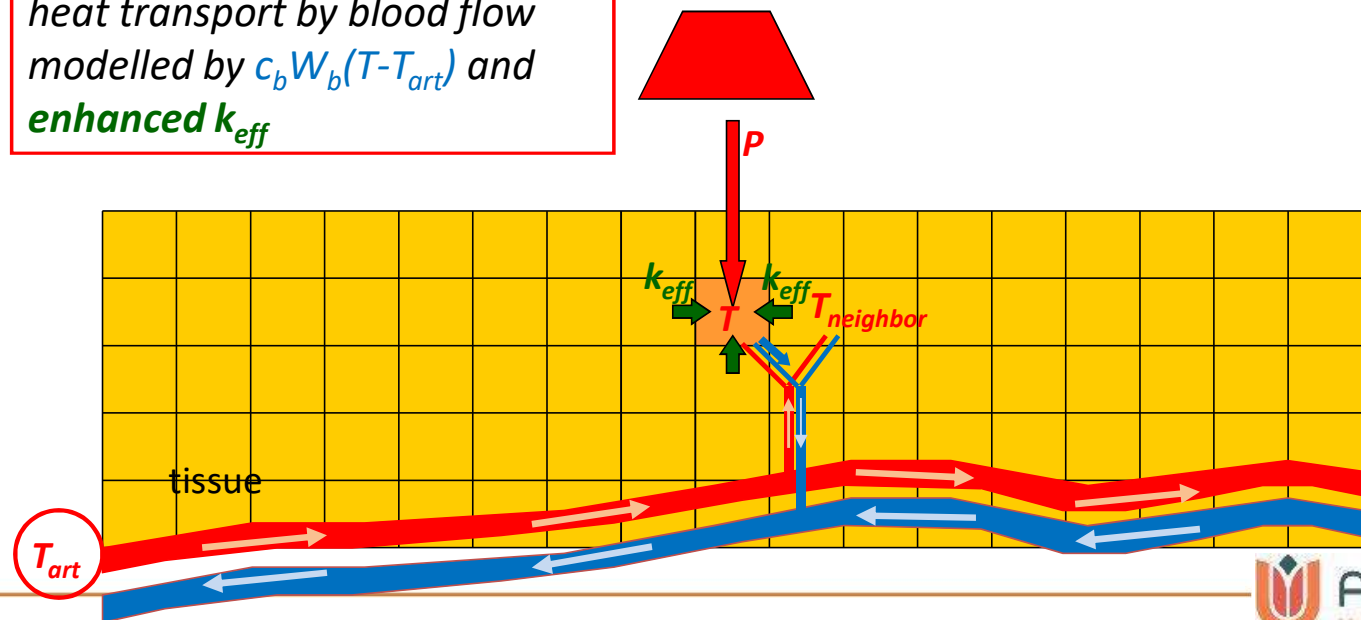
Crezee et al *Phys Med Biol* 39 813-822 (1994)

Heat transport in tissue

$$c\rho \frac{\partial T}{\partial t} = \nabla \cdot (k_{eff} \nabla T) - c_b W_b (T - T_{art}) + P$$

- Temperature change $c\rho \frac{\partial T}{\partial t}$
- Power density P
- Conduction $\nabla \cdot (k_{eff} \nabla T)$
- Perfusion W_b

heat transport by blood flow modelled by $c_b W_b (T - T_{art})$ and enhanced k_{eff}



Crezee et al *Phys Med Biol* 39 813-822 (1994)

Heat transport in tissue

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Discussed later

Heat transport in tissue

- Bio-heat equation
- Thermal properties of tissues
- Vascular cooling

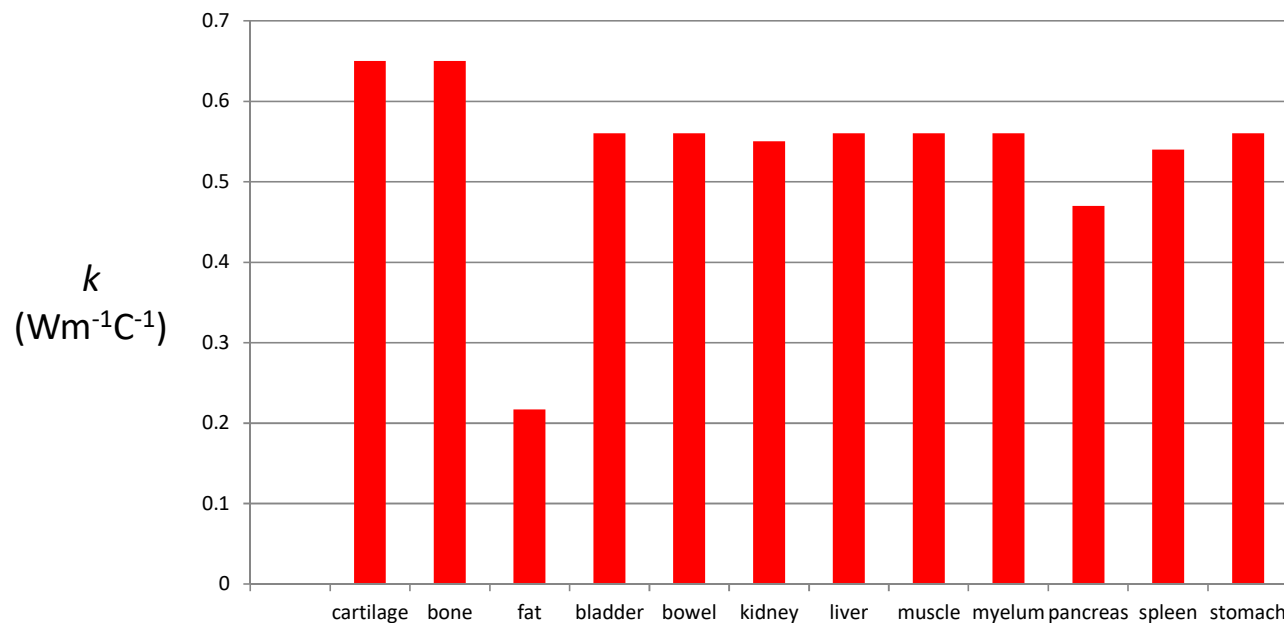
Thermal properties of tissue

Relevant thermal properties:

- Conduction
- Blood flow

Thermal properties of tissue

Thermal conduction:



ESHO task group committee *Treatment planning and modelling in hyperthermia* (1992)

Thermal properties of tissue

Thermal conduction:



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Thermal properties of tissue

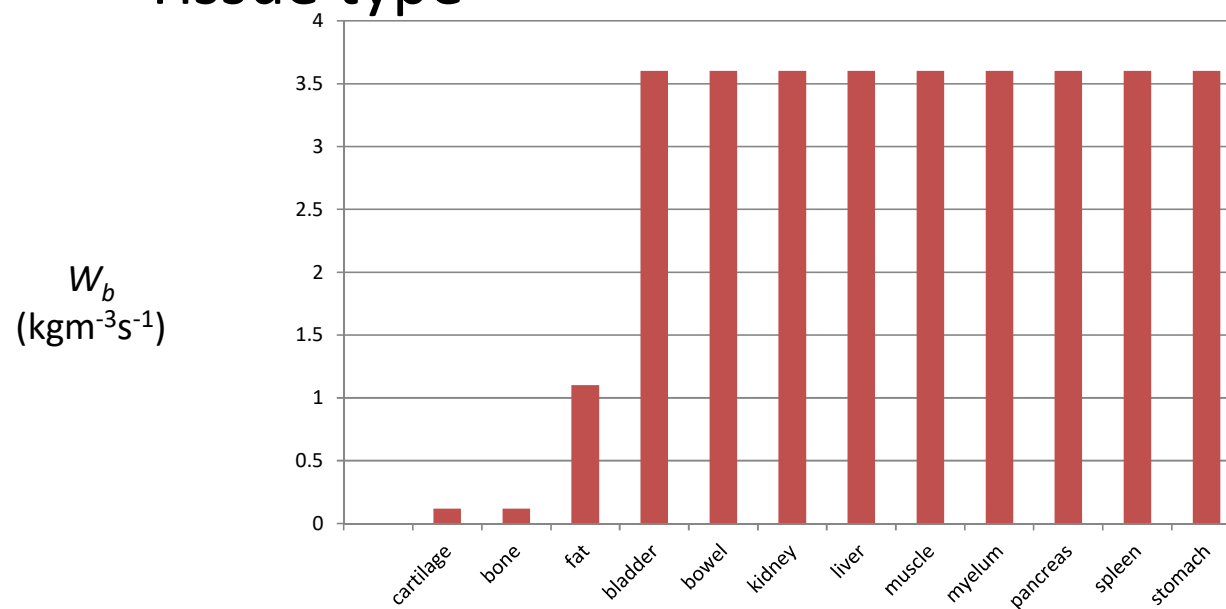
Blood flow depends on:

- Tissue type

Thermal properties of tissue

Blood flow depends on:

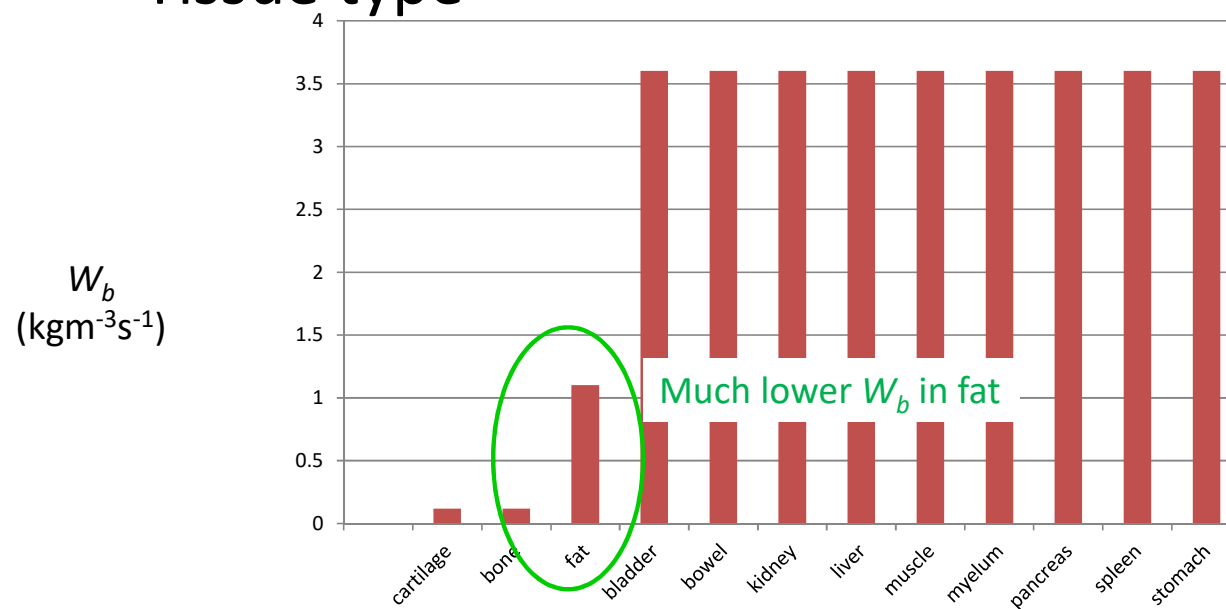
- Tissue type



Thermal properties of tissue

Blood flow depends on:

- Tissue type



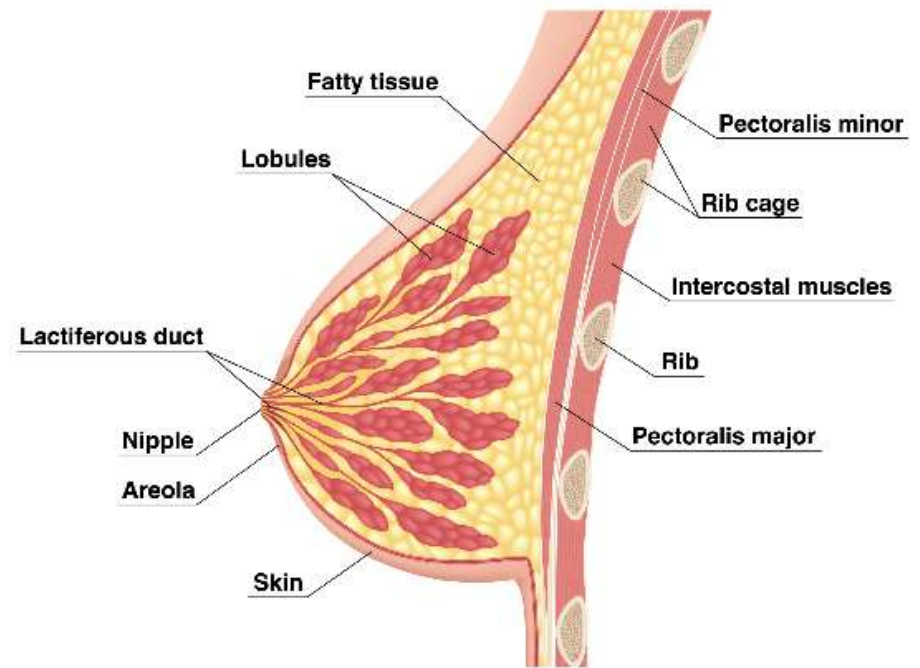
Thermal properties of tissue

Blood flow depends on:

- Tissue type
- Detailed data in recent reviews

ω_t (kg/s/m³) of healthy and tumour breast tissue.

Fibroglandular	Fat	Tumour Tissue
Min. Max.	Min. Max.	Min. Max.
0.189-0.754	0.014-8.798	0.530-22.260



Thermal properties of tissue

Blood flow depends on:

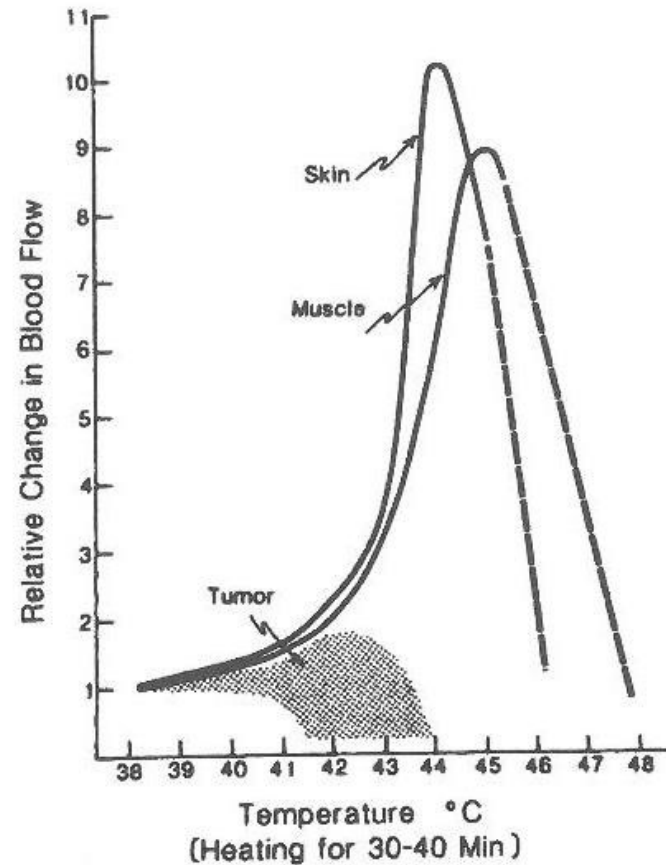
- Tissue type
- Detailed data in recent reviews
 - ITIS database popular

<https://itis.swiss/virtual-population/tissue-properties/database/>

Thermal properties of tissue

Blood flow depends on:

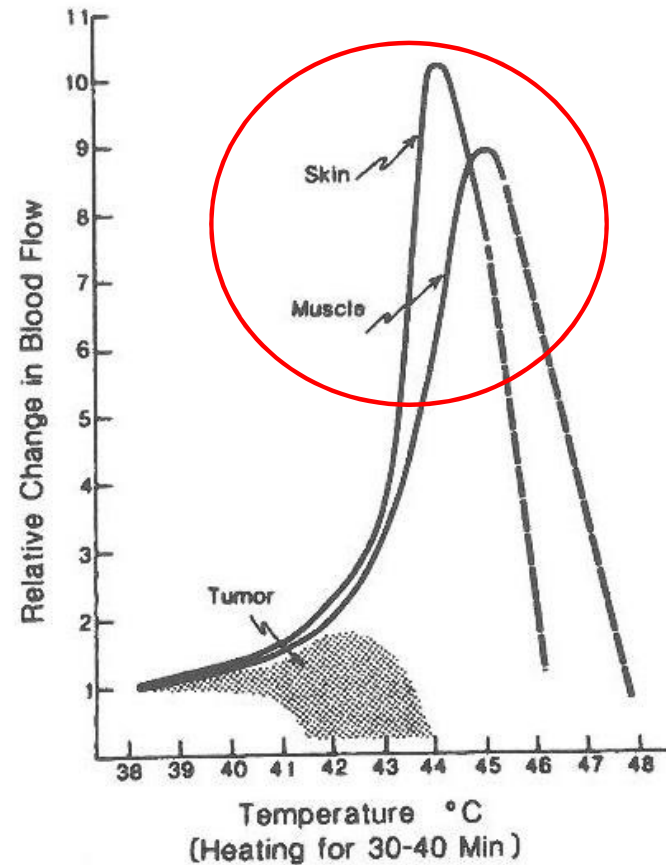
- Tissue type
- Temperature



Thermal properties of tissue

Blood flow depends on:

- Tissue type
- Temperature

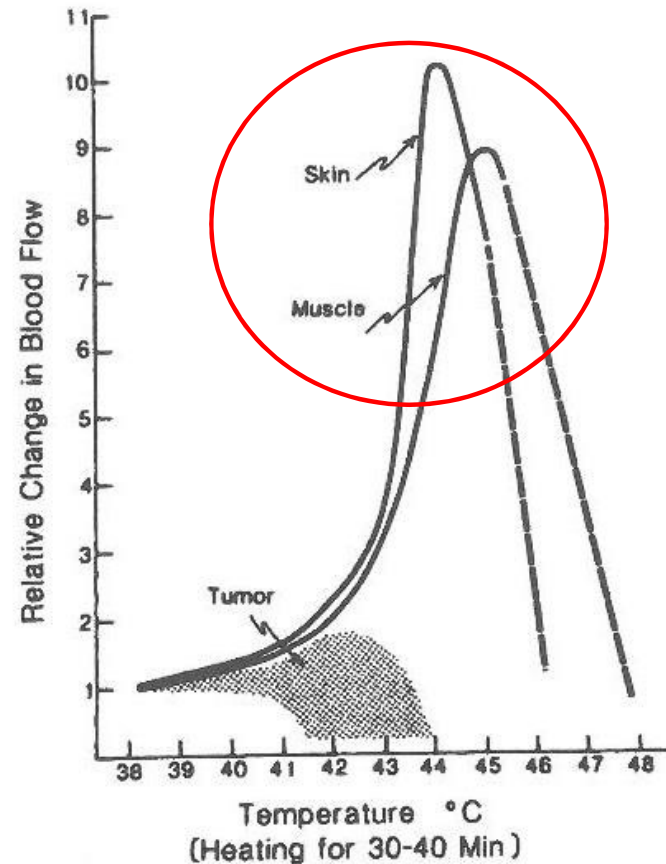


Thermal properties of tissue

Blood flow depends on:

- Tissue type
- Temperature

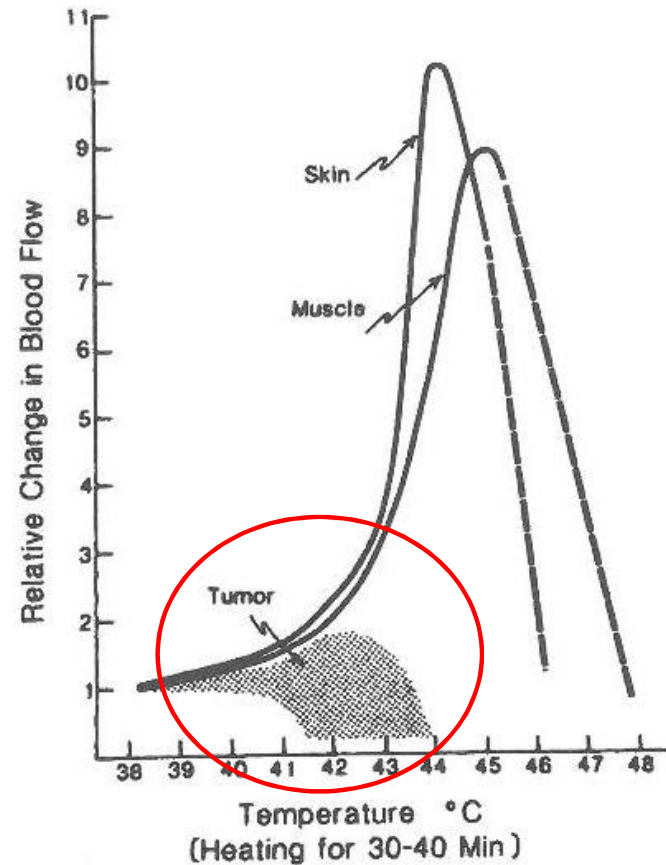
published perfusion data
are generally valid under
normothermic conditions



Thermal properties of tissue

Blood flow depends on:

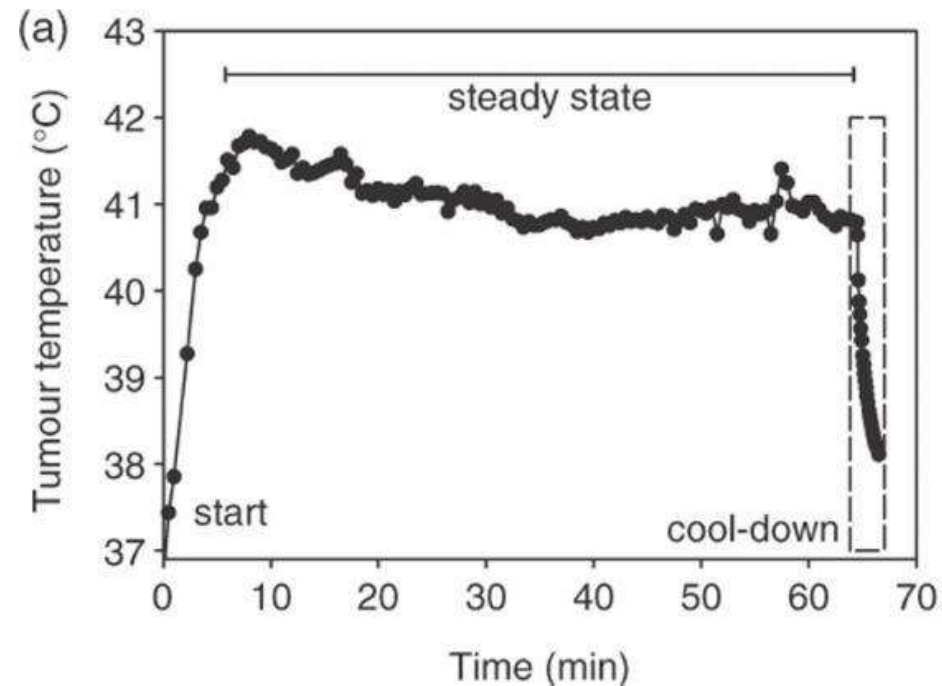
- Tissue type
- Temperature



Thermal properties of tissue

Blood flow depends on:

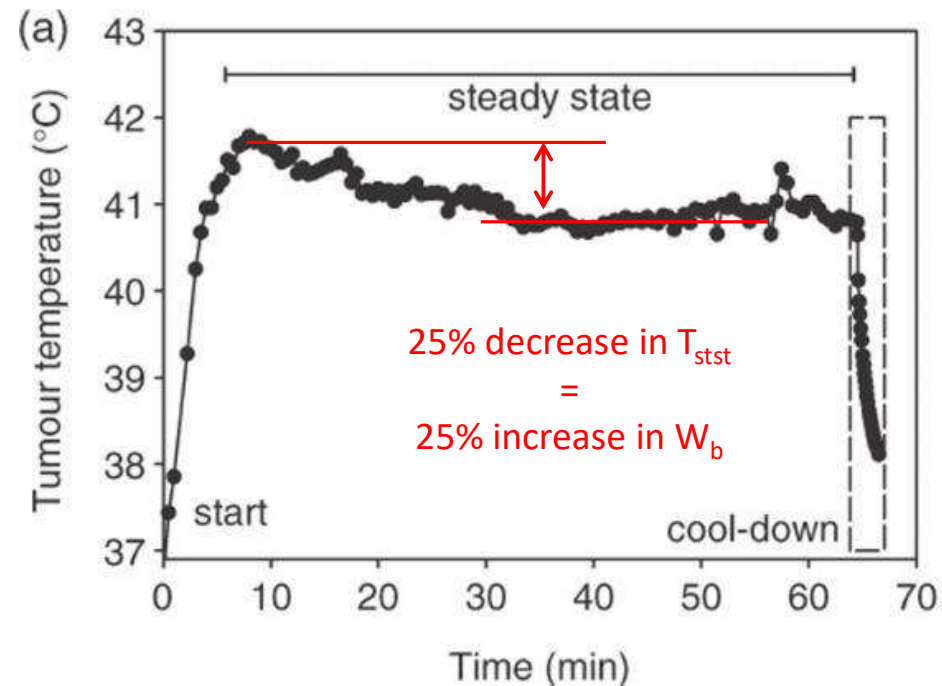
- Tissue type
- Temperature
- Time



Thermal properties of tissue

Blood flow depends on:

- Tissue type
- Temperature
- Time



Heat transport in tissue

- Bio-heat equation
- Thermal properties of tissues
- **Vascular cooling**

Vascular cooling

Impact of blood flow depends on vessel size:

Vascular cooling

Impact of blood flow depends on vessel size:

- **Small vessels:**
 - (almost) in thermal equilibrium with tissue
 - Modelled **collectively** with bio-heat equation

Vascular cooling

Impact of blood flow depends on vessel size:

- **Small vessels:**
 - (almost) in thermal equilibrium with tissue
 - Modelled **collectively** with bio-heat equation
 - Pennes bio-heat equation
 - Effective tissue conductivity

Vascular cooling

Impact of blood flow depends on vessel size:

- **Small vessels:**
 - (almost) in thermal equilibrium with tissue
 - Modelled collectively with bio-heat equation
- **Large vessels:**
 - Arteries cause cold tracks in tissue
 - Modelled **individually**

Vascular cooling

Cold track along large, unequilibrated vessels

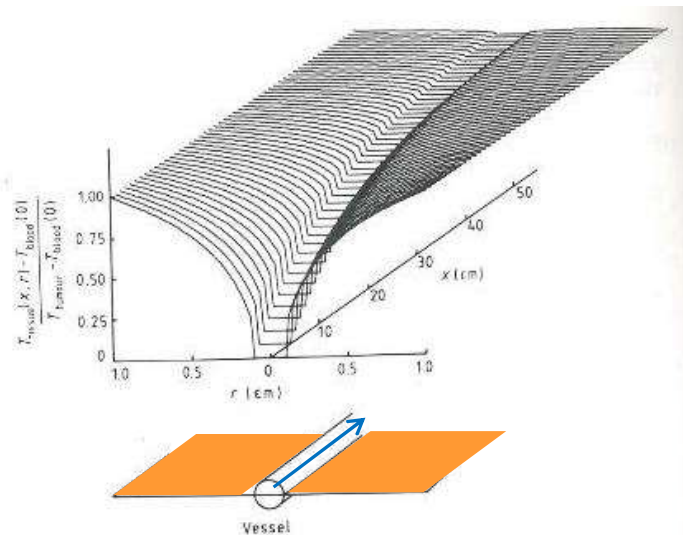


Figure 3. Two-dimensional plot of the temperature distribution in a plane through the axis of vessel 2 along the x -axis. Note the difference in scale between the x -axis and the r -axis (length x -axis 0.5 m).

Vascular cooling

Cold track along large, unequilibrated vessels

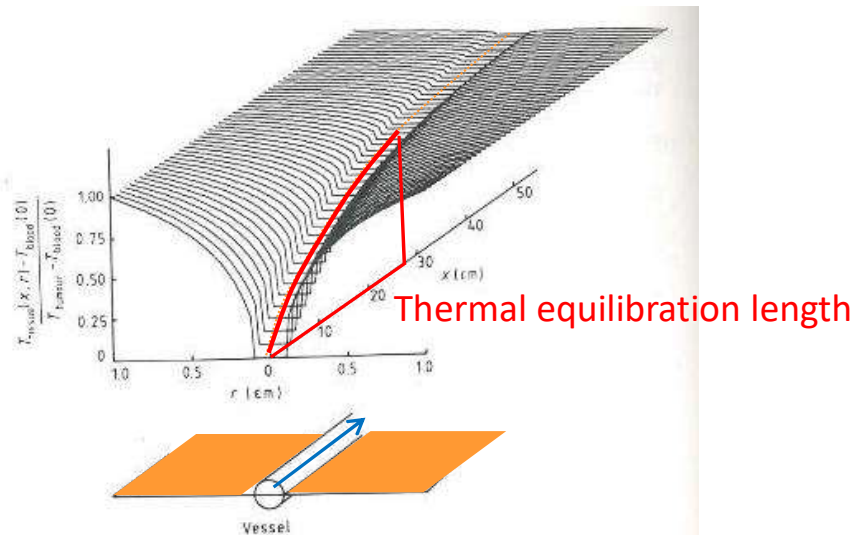


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Vascular cooling

Thermal equilibration length

Table 1. Typical vessel parameters for the circulation of a 13-kg dog. First four columns from Green (1950), based on Mall (1888). The Reynolds number, entrance and equilibration length computed with $\rho_b = 10^3 \text{ kgm}^{-3}$, $\mu = 0.0035 \text{ kgm}^{-1} \text{ s}^{-1}$, $c_b = 4 \times 10^3 \text{ Jkg}^{-1} \text{ K}^{-1}$, $Co_{10} = 0.1$, $Nu = 4.01$, $k_b = k_{eff} = 0.6 \text{ WK}^{-1} \text{ m}^{-1}$ and $b/a = 10$.

Vessel type	Diameter (mm)	Length (cm)	Velocity (cm s ⁻¹)	Number	Reynolds number	Entrance length (cm)	Equilibration length (cm)
Aorta	10	40	50	1	1400	830	11 700
Large arteries	3	20	13	4×10	110	20	270
Main branches	1	10	8	6×10^2	23	1.3	19
Secondary branches	0.6	4	8	10^3	14	0.5	6.7
Tertiary branches	0.14	1.4	3.4	7.6×10^4	1	0.01	0.16
Terminal branches	0.05	0.1	2	1×10^6	0.3	0.001	0.012
Terminal arteries	0.03	0.15	0.4	1.3×10^7	0.03	0.000 06	0.000 8
Arterioles	0.02	0.2	0.3	4×10^7	0.02	0.000 02	0.000 3
Capillaries	0.008	0.1	0.07	1.2×10^9	0.002	0.000 000 7	0.000 01
Venules	0.03	0.2	0.07	8×10^7	0.006	0.000 01	0.000 15
Terminal branches	0.07	0.15	0.07	1.3×10^7	0.014	0.000 06	0.000 8
Terminal veins	0.13	0.1	0.3	1×10^6	0.11	0.000 8	0.012
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Main veins	2.4	10	1.5	6×10^2	10	1.4	20
Large veins	6	20	3.6	4×10	60	22	300
Vena cava	12.5	40	33	1	1200	860	12 000

Crezee and Lagendijk *Phys Med Biol* 37 1321-1337 (1992)

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not in thermal equilibrium

Vascular cooling

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Terminal branches	0.05	0.1	2	1 × 10 ⁶	0.3	0.001	0.012
Terminal arteries	0.03	0.15	0.4	1.3 × 10 ⁷	0.03	0.000 06	0.000 8
Arterioles	0.02	0.2	0.3	4 × 10 ⁷	0.02	0.000 02	0.000 3
Capillaries	0.008	0.1	0.07	1.2 × 10 ⁹	0.002	0.000 000 7	0.000 01
Venules	0.03	0.2	0.07	8 × 10 ⁷	0.006	0.000 01	0.000 15
Terminal branches	0.07	0.15	0.07	1.3 × 10 ⁷	0.014	0.000 06	0.000 8
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Main veins	2.4	10	1.5	6 × 10 ²	10	1.4	20
Large veins	6	20	3.6	4 × 10	60	22	300
Vena cava	12.5	40	33	1	1200	860	12 000

modelled individually

Discrete vessels

not in thermal equilibrium

Vascular cooling

Thermal equilibration length

Table 1. Typical vessel parameters for the circulation of a 13-kg dog. First four columns from Green (1950), based on Mall (1888). The Reynolds number, entrance and equilibration length computed with $\rho_b = 10^3 \text{ kgm}^{-3}$, $\mu = 0.0035 \text{ kgm}^{-1} \text{ s}^{-1}$, $c_b = 4 \times 10^3 \text{ Jkg}^{-1} \text{ K}^{-1}$, $Co_{10} = 0.1$, $Nu = 4.01$, $k_b = k_{eff} = 0.6 \text{ WK}^{-1} \text{ m}^{-1}$ and $b/a = 10$.

Vessel type	Diameter (mm)	Length (cm)	Velocity (cm s ⁻¹)	Number	Reynolds number	Entrance length (cm)	Equilibration length (cm)
Aorta	10	40	50	1	1400	830	11 700
Large arteries	3	20	13	4 × 10 ³	110	20	270
Main branches	1	10	8	6 × 10 ²	23	1.3	19
Secondary branches	0.6	4	8	10 ³	14	0.5	6.7
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Crezee and Lagendijk *Phys Med Biol* 37 1321-1337 (1992)

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continuum model

Pennes or k_{eff}

← near thermal equilibrium

Vascular cooling

Impact of blood flow depends on vessel size:

- **Small vessels:**
 - (almost) in thermal equilibrium with tissue
 - Modelled **collectively** with bio-heat equation

Vascular cooling

Impact of blood flow depends on vessel size:

- **Small vessels:**
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- **Large vessels:**
 - Arteries cause cold tracks in tissue
 - Modelled **individually**

Vascular cooling

Impact of blood flow depends on vessel size:

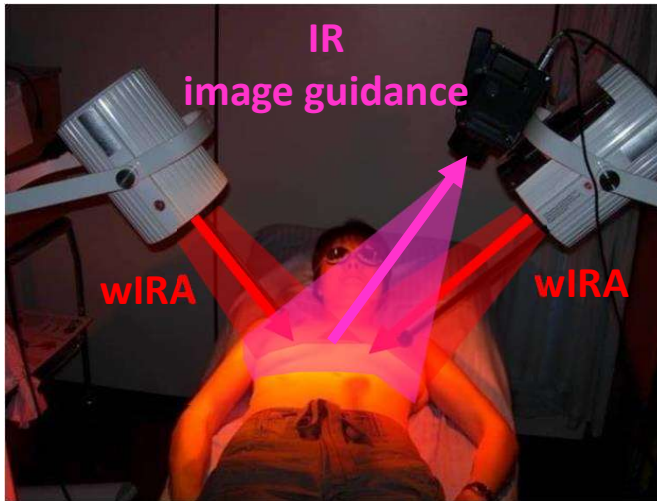
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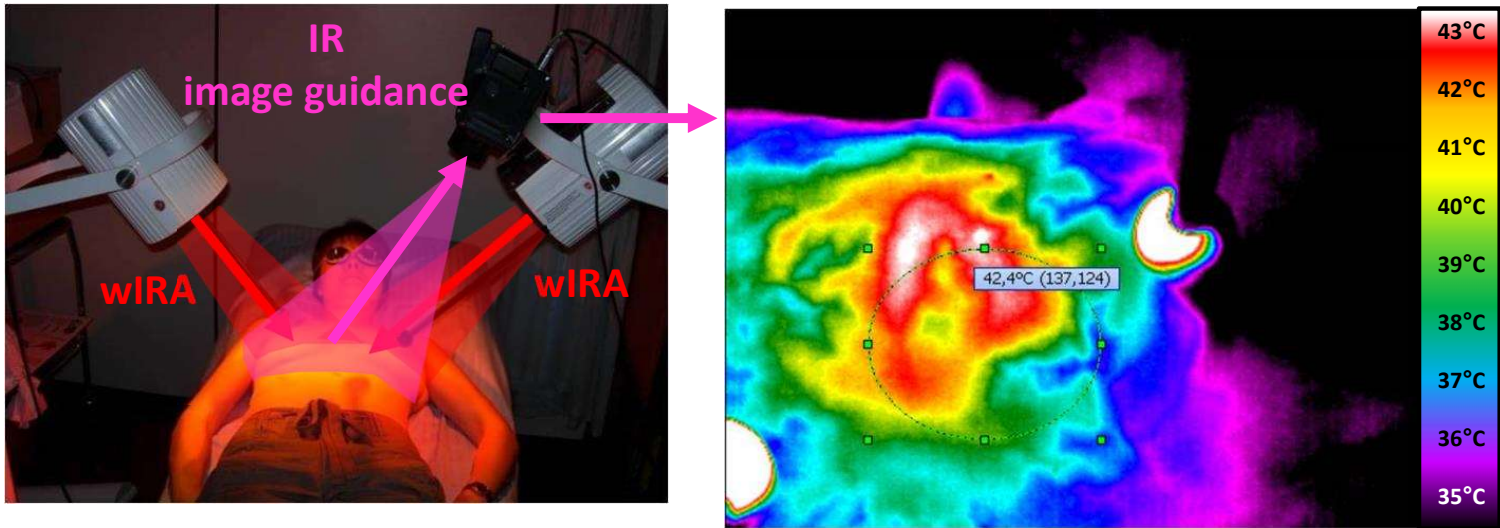
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Vascular cooling



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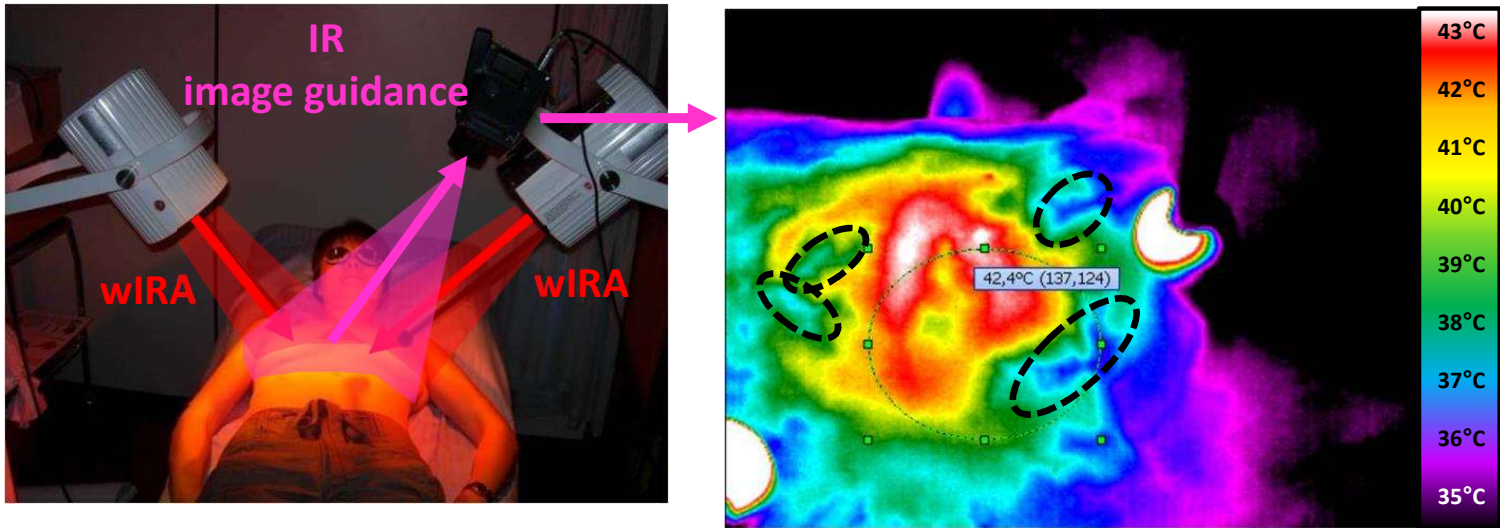
Vascular cooling



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Vascular cooling



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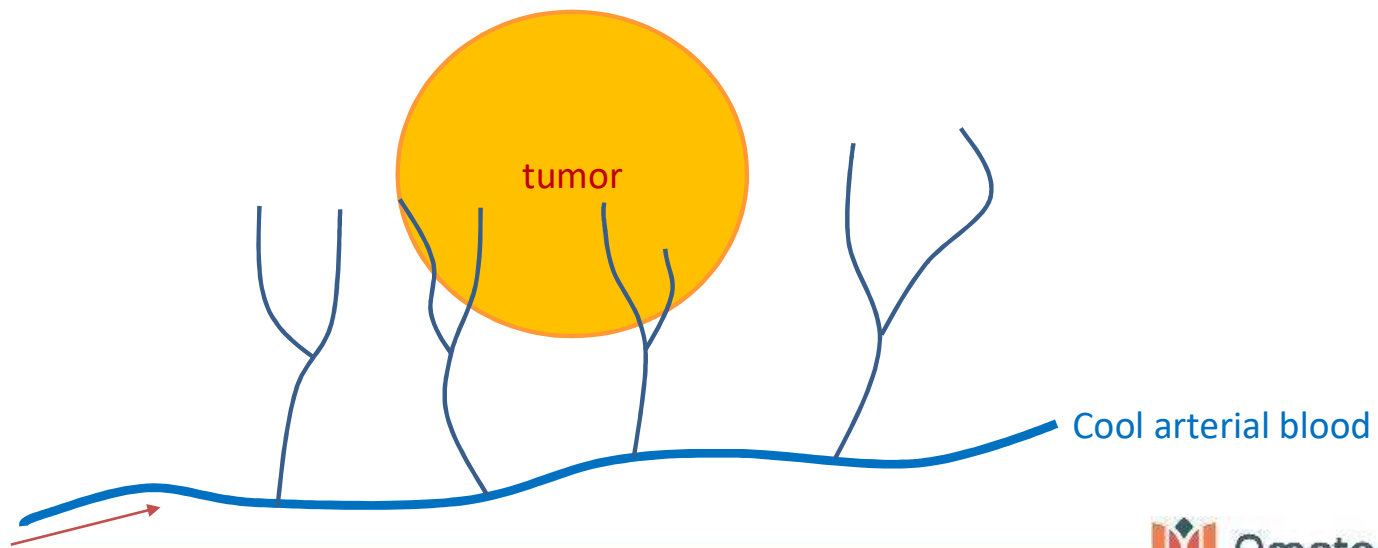
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Pre-heat arteries by heating
Large margin around tumor

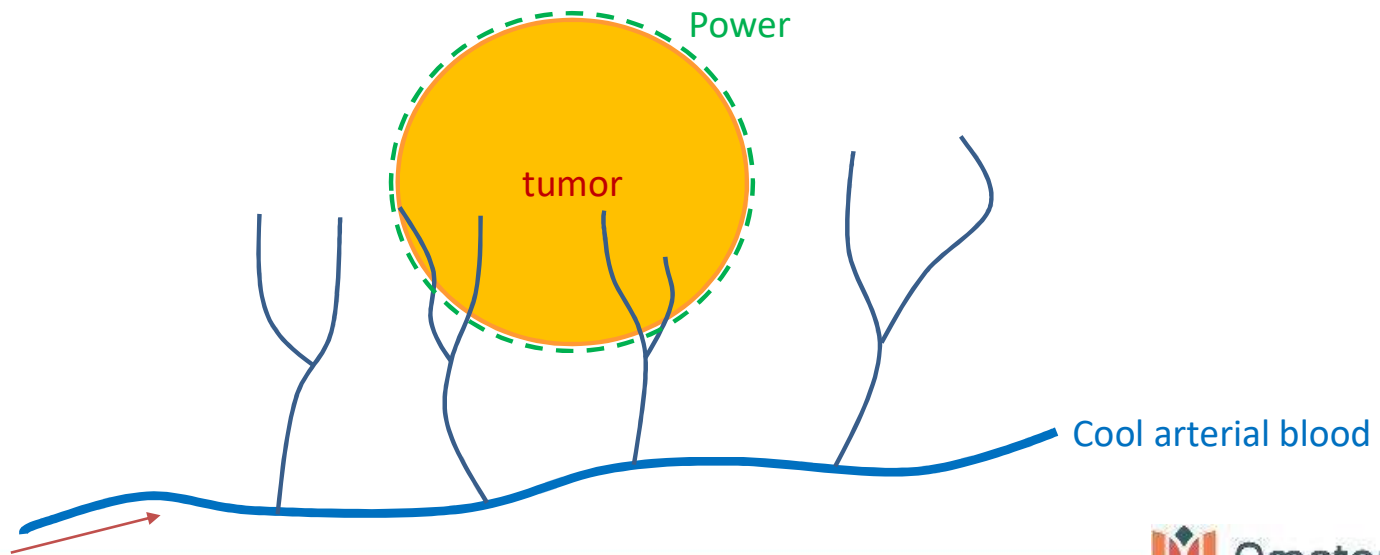
Vascular cooling

Preheating arterial blood flow



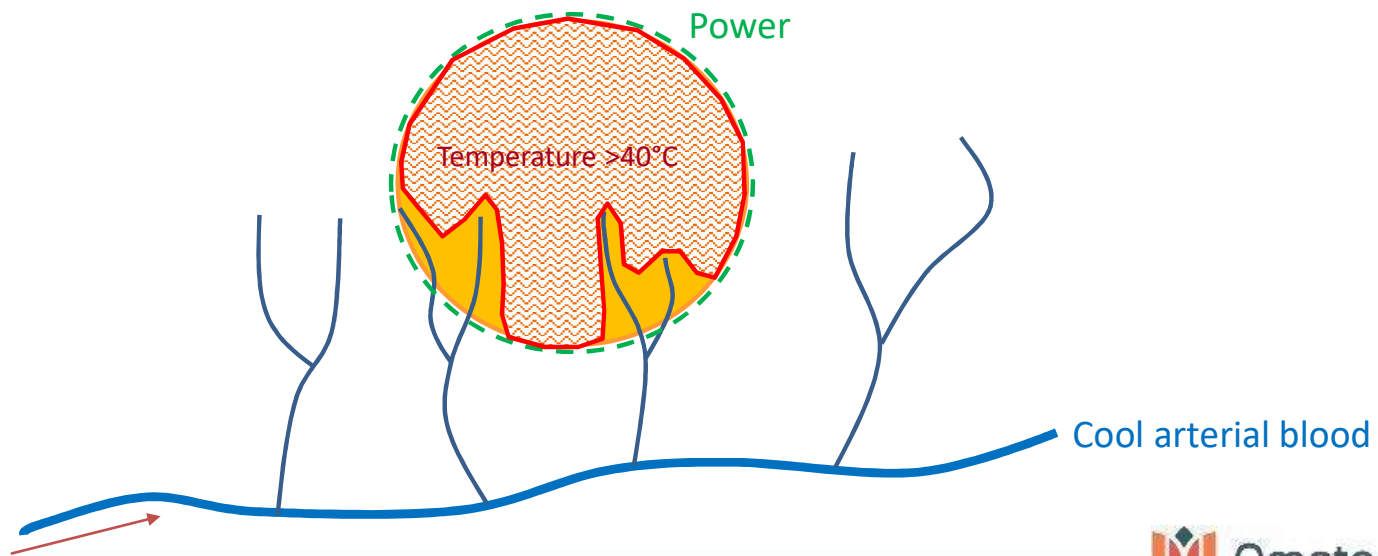
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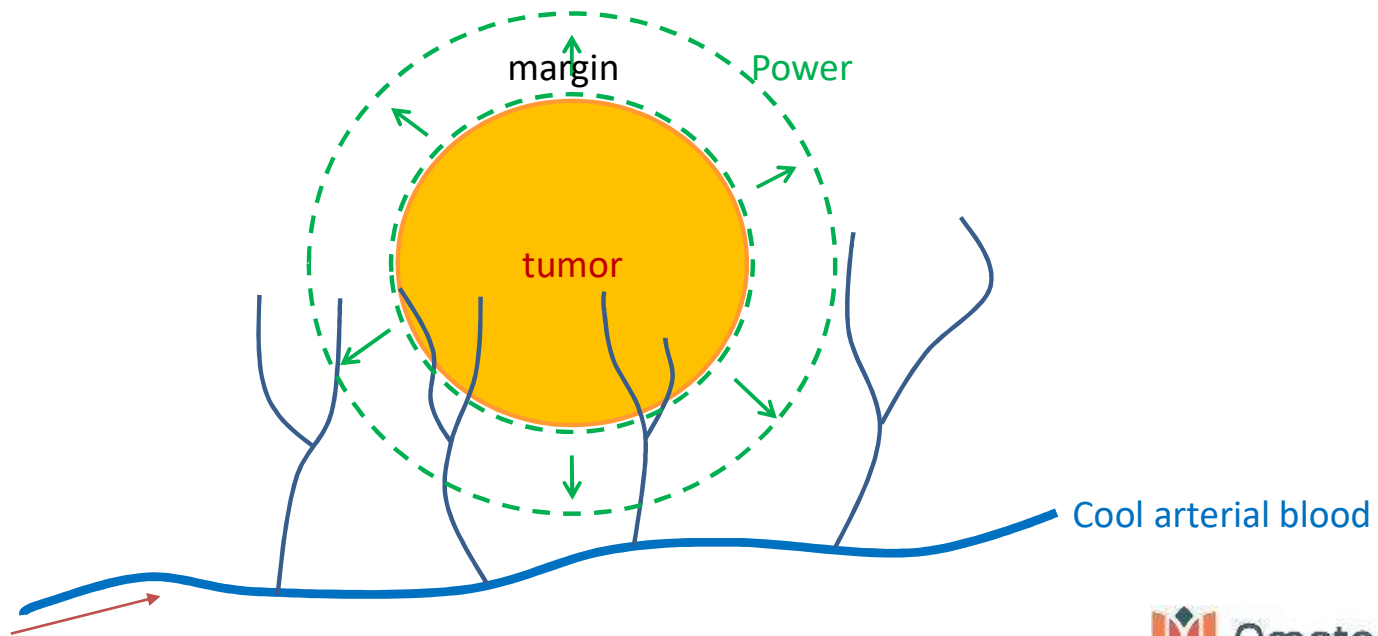
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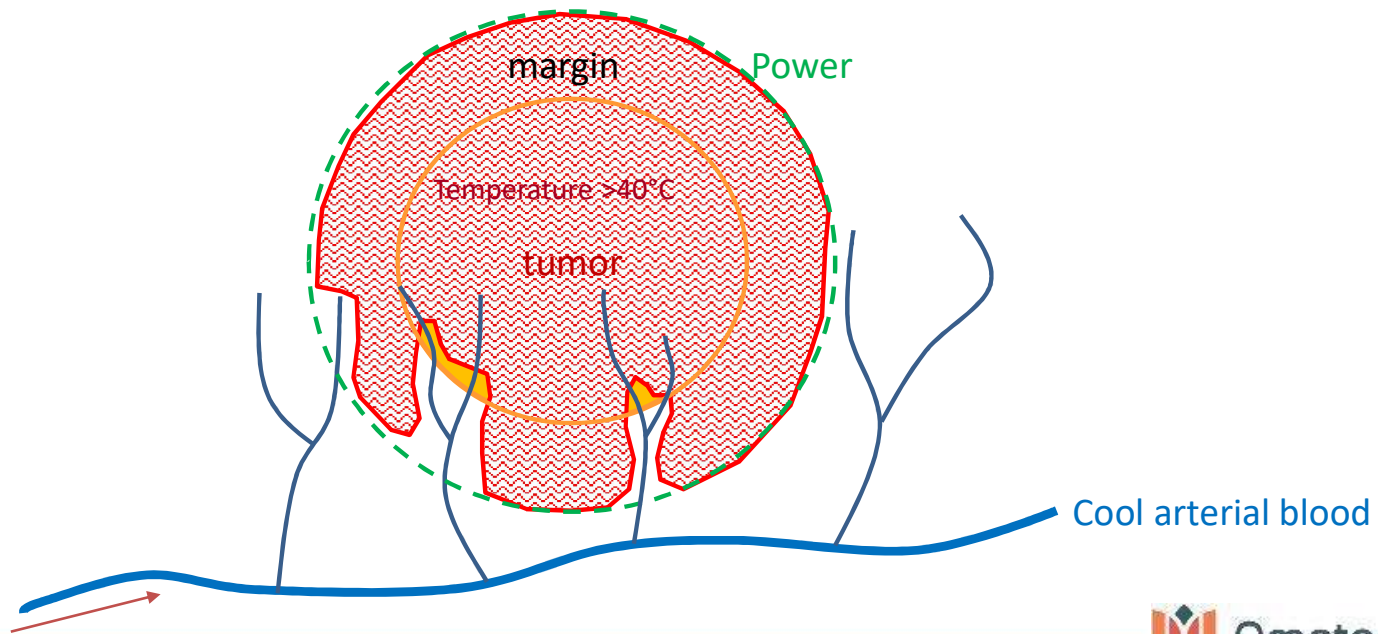
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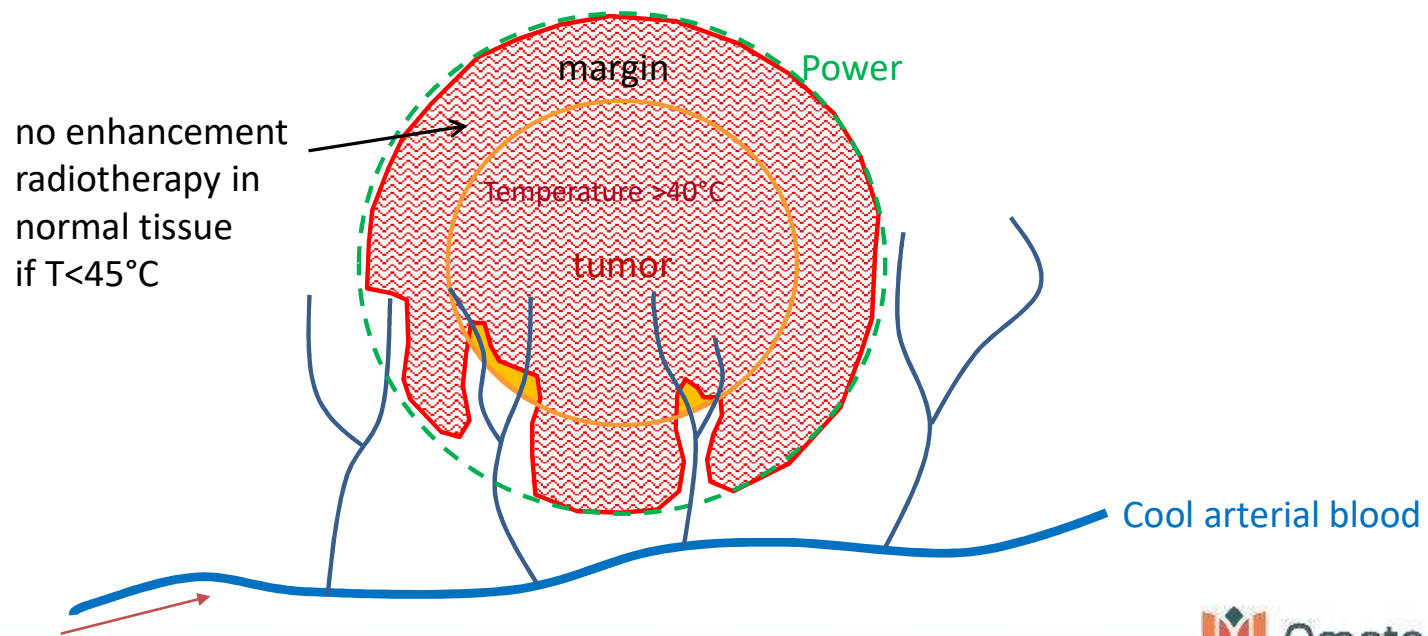
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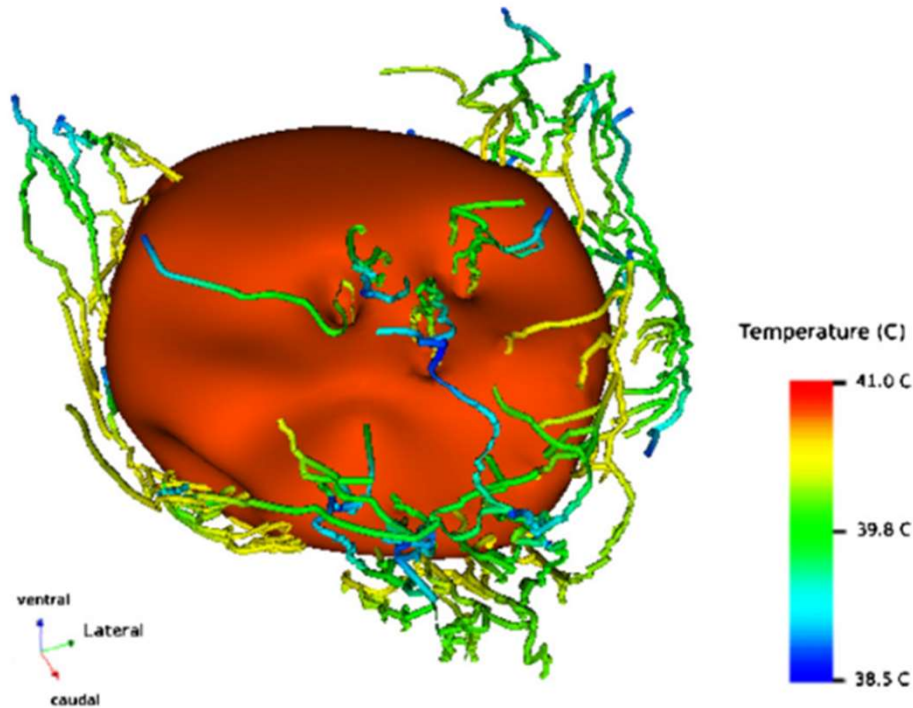
Vascular cooling

Preheating arterial blood flow



Vascular cooling

Example: locoregional hyperthermia of prostate



Van den Berg *et al. Phys Med Biol.* 2006; 51: 809-825

Heat transport in tissue

- Bio-heat equation
- Thermal properties of tissues
- Vascular cooling

Heat transport in tissue

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Conclusion: bio heat transfer is a complex and challenging topic, with limited accuracy for Pennes equation

Heat transport in tissue

- Bio-heat equation
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- Vascular cooling

Tomorrow:

modeling heat transport in treatment planning

Heat transport in tissue

Thank you for your attention!

Hans Crezee

Questions?

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Amsterdam UMC
University Medical Centers